Candidacy Exam — Spring 2002

QUANTUM MECHANICS

Total Points: 100

Attempt all the problems. There are several parts to each problem with unequal weights.

Useful Information:

In spherical coordinates,

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

In cylindrical coordinates,

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}$$

1. [30 points]

Consider the following operators in a 3-dimensional Hilbert space spanned by the *orthonormal* basis formed by the kets $|u_i>$; $i = 1, 2, 3$:

$$\hat{A} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$\hat{W} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The physical state is given by:

$$|\psi> = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}.$$

[10] (a) If $\hat{A}$ is measured, what <u>values</u> can be obtained? With what <u>probabilities</u>?

[8] (b) If $\hat{A}$ is measured in the state $|\psi>$, and a *value* $+1$ is obtained, what is the <u>state after</u> the measurement?

[12] (c) If, *after* the measurement of $\hat{A}$ above in (b), $\hat{W}$ is measured, what <u>values</u> would be obtained? With what <u>probabilities</u>?

2. [35 points]

Consider a particle of mass m in one dimension subjected to the following singular attractive potential:

$$V(x) = -\frac{\hbar^2}{2m}\ \lambda\ \delta(x-a)$$

where λ and a are constants.

[10] (a) For this potential, is the wavefunction continuous in all regions of the 1-dimensional space? Is the slope of the wavefunction continuous in all regions of the 1-dimensional space? Show how you arrive at your conclusions.

[15] (b) Considering energies appropriate to the scattering state problem, obtain the transmission coefficient for the particle.

[10] (c) Based on what you know about *poles* of the transmission amplitude in the complex plane and bound states in the same potential, calculate the bound state energy of the particle in this potential.

3. [35 points]

A particle of mass m is subjected to a potential V, and constrained to move on the surface of a *sphere* of radius a. Its momentum operator in 3-dimensions is: $\mathbf{P} = -i\hbar\nabla$.

[10 p] (a) Choosing the center of the sphere as the origin, show that the Hamiltonian of the particle is given by

$$H = \frac{\mathbf{L^2}}{2I} + V(\theta, \phi)$$

where $\mathbf{L^2}$ is the 3-D angular momentum operator and $I = ma^2$ is the moment of inertia of the particle with respect to the origin.

Why is V *independent* of r in this case ?

[15 p] (b) Now assume that the particle is *free* except that it is still constrained to move on the surface of the sphere.

Obtain the energy eigenvalues and eigenstates of the particle.

What is the degeneracy of the energy levels. Explain the reason for the degeneracy.

[10 p] (c) If instead of the sphere, the particle was constrained to move *freely* on the surface of a *cylinder* of radius a, what would be the form of the eigenstate(s) ? Explain your answer.