Quantum and Atomic Physics

1. Consider a Bohr orbit with a very large radius, say $r_B = 1$ mm, in a hydrogen-like atom with $Z = 25$. [Note: $E_n = \mathcal{R}\frac{Z^2}{n^2}$, where $\mathcal{R}$=13.6 eV, and the first Bohr radius of hydrogen is $a_0 = 5.29 * 10^{-9}$ cm.]

(10 p) (a) Estimate the corresponding quantum number, n.

(10 p) (b) Assume that a transition between two neighboring quantum states occurs in this energy region ($\Delta n = 1$). Calculate the energy of the emitted radiation.

(25 p) 2. Let $|a'\rangle$ and $|a''\rangle$ be the properly normalized eigenkets of a Hermitian operator $\hat{A}$ with eigenvalues a' and a'', respectively ($a' \neq a''$). The Hamiltonian is given by

$$\hat{H} = |a'\rangle\delta\langle a''| + |a''\rangle\delta\langle a'| \,, \tag{1}$$

where δ is a real number. Determine the eigenvalues and eigenstates of the Hamiltonian.

3. Consider a beam of neutrons, each of which has momentum $\hbar k_0$. The beam is "chopped," producing the following wave function for each neutron at the instant after the preparation of the chopped beam ($t_0 = 0$):

$$\psi(x) = \begin{cases} \frac{1}{\sqrt{L}}\exp(ik_0 x) & \text{if } -\frac{L}{2} \leq x \leq +\frac{L}{2} \\ 0 & \text{elsewhere} \end{cases} \,. \tag{2}$$

The momentum of a neutron is now measured.

(15 p) (a) What values can be found and with what probability do these values occur?

(10 p) (b) Sketch the probability density as a function of the wave number. What momentum values have zero probability to be found?

(10 p) (c) What is the wave function at a later time, t? What are the probabilities of the different momentum values to be measured at time t?

(20 p) 4. At a given instant of time, the angle-dependent part of the wave function of a particle is known to be

$$\chi(\theta, \phi) = \left(\frac{3}{4\pi}\right)^{1/2} \sin\phi \sin\theta \ . \tag{3}$$

What possible values of $\hat{L}^2$ and $\hat{L}_z$ will measurement find, and with what probabilities will these values occur? [The first few spherical harmonics are given by

$$\begin{aligned}
Y_0^0 &= \left(\frac{1}{4\pi}\right)^{1/2} \\
Y_1^0 &= \frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2} \cos\theta \\
Y_1^1 &= -\frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} \sin\theta e^{i\phi} \\
Y_1^{-1} &= \frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} \sin\theta e^{-i\phi} \ .]
\end{aligned}$$