Candidacy Exam – Fall 2010

Quantum Exam

1.) Assume a particle of mass m in a one dimensional system with a potential V(x) = a|x| where a is a constant. Also assume a trial normalized wave function ψ:

$$\psi = \left(\frac{2b}{\pi}\right)^{1/4} e^{-bx^2},$$

where b is a parameter ≥ 0.

(10 points) a) Is the given ψ an eigenstate of the Hamiltonian H?

(20 points) b) For the given ψ, calculate the expectation value of H.

(10 points) c) Let E_G be the notation for the ground state energy of the system and E_U be an upper bound, that is $E_G \leq E_U$. Determine the lowest value of E_U you can find using ψ.

Possibly useful integrals:

n is a positive integer

$$\int_0^\infty dx\, x^{2n} e^{-x^2} = \frac{1\,(3)\ldots(2n-1)\sqrt{\pi}}{2^{n+1}}$$

$$\int_0^\infty dx\, x^{2n+1} e^{-x^2} = \frac{n!}{2}$$

$$\int_0^\infty dx\, e^{-x^2} = \frac{1}{2}\sqrt{\pi}$$

2.) The spin-orbit operator is $2\vec{L}\cdot\vec{S}$. Let $\vec{J} = \vec{L} + \vec{S}$, where $\vec{L}$ is orbital angular momentum, $\vec{S}$ is spin, and $\vec{J}$ is total angular momentum.

(15 points) a) Show that $[S^2, \vec{L}\cdot\vec{S}] = [L^2, \vec{L}\cdot\vec{S}] = [J^2, \vec{L}\cdot\vec{S}] = 0$.

(10 points) b) Derive the eigenvalues of the $2\vec{L}\cdot\vec{S}$ operator.

3.) In a representation where the eigenspinors of the $\hat{S}_z$ operator are the basis vectors $\binom{1}{0}$ and $\binom{0}{1}$, the $\hat{S}_z$ operator is $\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ and the $\hat{S}_x$ operator is $\frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$.

(10 points) a) Find the eigenvalues and normalized eigenspinors of $\hat{S}_X$.

(10 points) b) In an initial state of $\chi = \begin{pmatrix}\sqrt{3}/2\\ 1/2\end{pmatrix}$, $\hat{S}_X$ is measured. What are the possible results, and their probabilities?

(10 points) c) Subsequently, $\hat{S}_z$ is measured. What are the possible results and their probabilities?

(5 points) d) What would have been the possible results and their probabilities if only $\hat{S}_z$ was measured (i.e. $\hat{S}_X$ had not been measured) ?