E & M

For full credit, you must answer all questions.

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1. (25 points) Two unknown but identical charges, separated by a distance d, have a grounded spherical conductor placed midway between them. What should be the approximate radius R of the sphere in order for the two unknown charges to experience no net force?
   Hints: You should use the approximation that R/d is small and neglect higher powers of R/d. Also, you may assume that for a charge q at a distance $d/2$ from the center of a grounded conducting sphere of radius R, the image charge has magnitude $-2Rq/d$ and is located a distance $2R^2/d$ from the center of the sphere.

2. A spherically symmetric, static charge distribution generates an electric field of the form

$$\mathbf{E}(\mathbf{r}) = k_1 \frac{e^{-k_2 r}}{r^3}\mathbf{r}$$

   where k_1 and k_2 are positive constants.

   (*a*) (15 points) What is the total electric charge?

   (*b*) (10 points) Is there a point charge at the origin? If so, what is its sign and magnitude?

3. A battery is connected to a resistor via a coaxial cable, whose resistance can be neglected. The cable has an inner conductor of radius a and an outer cylindrical shield of internal radius b. The voltage between the two conductors of the cable is V, and the current in the circuit is I.

   (*a*) (6 points) Find the magnetic field as a function of radial distance s from the center of the inner conductor, for $a < s < b$.

   (*b*) (9 points) Derive an expression for the electric field in the same region, as a function of V.

   (*c*) (5 points) Calculate the Poynting Vector for $a < s < b$.

   (*d*) (5 points) Use your expression for the Poynting Vector to show that the power carried by the cable is $P = VI$.

4. (*a*) (9 points) Write expressions for the electric and magnetic field vectors for an electromagnetic plane wave of angular frequency ω, with electric field amplitude E_0, zero fields at the origin at time $t = 0$, vertically polarized, and propagating 30° west of north (take $\hat{\mathbf{x}}$ pointing east, $\hat{\mathbf{y}}$ pointing north and $\hat{\mathbf{z}}$ vertically upward). **Note:** be sure to express the vectors $\mathbf{E}$ and $\mathbf{B}$ as well as the wave vector *explicitly* in terms of Cartesian components.

   (*b*) (9 points) Next, write $\mathbf{E}$ and $\mathbf{B}$ for a similar wave except that now its propagation direction points 45° above the horizon, and 45° east of north, and the polarization has zero component along $\hat{\mathbf{y}}$.

   (*c*) (2 points) To approximately what level of accuracy may we assume that the properties of air are the same as those of a vacuum with regard to propagation of EM waves? Specifically, if we write $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$ for air, is the error in the worst of these two cases 10% or more, 1 to 10%, 0.1% to 1%, or less than 0.1%?

   (*d*) (5 points) Define phase velocity and group velocity, and explain briefly under what situations can the phase and group velocities differ for an EM wave.