E & M

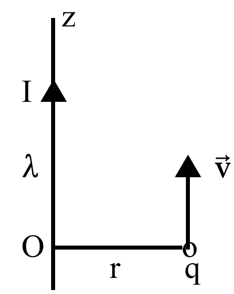
(25 pts.) 1. A sphere of radius R carries a charge density $\rho(r) = \kappa r$ (where κ is constant and r is the radial distance from the center of the sphere).

a) Calculate the electric field inside and outside the sphere.

b) Calculate the electric potential inside and outside the sphere.

c) Calculate the energy of this configuration.

(25 pts.) 2. A particle with charge q is traveling with velocity $\vec{v}$ parallel to a wire with a uniform linear charge distribution λ per unit length. The wire also carries a current I as shown in the figure. What must the velocity be for the particle to travel in a straight line parallel to the wire, a distance r away?



(25 pts.) 3. Assume that the existence of a magnetic charge is related to the magnetic field by the local relation: $\vec{\nabla} \cdot \vec{B} = \mu_o \rho_m$

a) Using the divergence theorem, obtain the magnetic field of a point magnetic charge at the origin.

b) Show that Faraday's law is incompatible with a magnetic charge density that is a function of time.

c) Assuming that the magnetic charge is conserved, derive the local relation between the magnetic charge current density $\vec{J}_m$ and the magnetic charge density ρ_m.

d) Modify Faraday's law to obtain a law consistent with the presence of a magnetic charge density that is a function of position and time. Demonstrate the consistency of the modified law.

(25 pts.) 4. Consider electromagnetic waves in free space of the form:

$$\vec{E} = \vec{E}_o(x,y)\, e^{i(kz - \omega t)}$$

$$\vec{B} = \vec{B}_o(x,y)\, e^{i(kz - \omega t)}$$

a) Find the relation between k and ω, as well as the relation between $\vec{E}_o$ and $\vec{B}_o$.

b) Show that $\vec{E}_o$ and $\vec{B}_o$ satisfy the equations for electrostatics and magnetostatics in free space.