Candidacy Exam **Spring 2006**

Electricity and Magnetism

Instructions: Work each of the following three problems. The total credit is 100 points.

1. A good electrical conductor is a material in which free electrons are mobile under the influence of an external electric field. Consider an electrical conductor in electrostatic equilibrium. Answer the following questions rigorously, using diagrams and Gauss' law or other equations, as needed.

   (a) (6 pts) Discuss the electric field inside such a conductor. Is the field zero or can it be nonzero? Explain your answer.

   (b) (7 pts) Suppose that an isolated conductor carries a net electric charge. Explain why the net charge must reside entirely on the surface of the conductor.

   (c) (7 pts) Explain why the surface of a conductor is an equipotential. Also discuss the electrostatic potential *inside* a conductor.

2. Consider electromagnetic fields inside of an arbitrary medium.

   (a) (12 pts) State the general differential form of Maxwell's equations in a medium. The equations should be expressed in terms of the fields $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{H}$.

   (b) (10 pts) Derive a relationship between the free charge and current densities in the medium (i.e., derive the charge continuity equation).

   (c) (18 pts) Suppose now that the medium is infinite, homogeneous, isotropic, source-free and nondispersive, so that $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$, where ϵ and μ are constants. In such a medium, a plane wave can propagate. If the plane wave has an electric field given by

   $$\mathbf{E} = \mathbf{E}_0 \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)} ,$$

   find the corresponding field $\mathbf{B}$ in terms of $\mathbf{k}$ and $\mathbf{E}$, and determine how the magnitude of $\mathbf{k}$ is related to ϵ, μ, and ω.

3. (40 pts) Two infinite, grounded metal plates lie parallel to the xz plane, one at $y = 0$, the other at $y = a$. The left end (see diagram) is closed

off with an infinite strip insulated from the two plates and maintained at constant potential V_0. Find the potential in the slot, expressed as an infinite series. The following integral might be useful:

$$\int_0^\pi \sin n\theta \ \sin m\theta \ d\theta = \frac{\pi}{2} \, \delta_{mn} \ ,$$

where m and n are integers.