# Electricity and Magnetism

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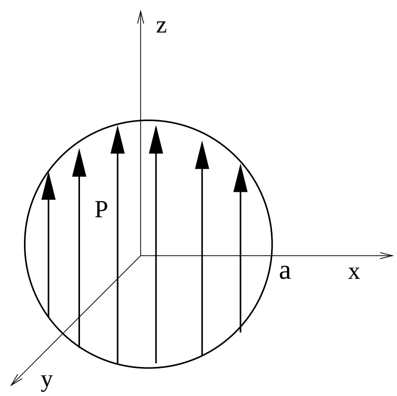
1. Consider sphere of radius $a$ centered at the origin with zero free charge and a uniform permanent polarization $\mathbf{P} = P\hat{\mathbf{z}}$.




[5 pts] (a) What is the bound volume charge density, $\rho_{\rm b}(\mathbf{r})$, and surface charge density, $\sigma_{\rm b}(\mathbf{r})$?

[5 pts] (b) Show that $\int \mathbf{E} \cdot d\mathbf{S}$ integrated over any spherical surface centered at the origin vanishes.

[15 pts] (c) Find the potential both inside and outside the sphere by solving Laplace's equation.

**Note**: Recall that solutions of Laplace's equation with azimuthal symmetry can be written as a series in Legendre polynomials

$$\phi(r,\theta) = \sum_{l=0}^{\infty} \left[ A_l r^l + \frac{B_l}{r^{l+1}} \right] P_l(\cos\theta).$$

2. Consider a medium that is linear and isotropic but not homogeneous in both conducting and dielectric properties. Let $\epsilon(\mathbf{r})$ be the permittivity and $\sigma(\mathbf{r})$ be the conductivity of the material.

[5 pts] (a) Write down the differential form of Gauss' Law for the displacement field, $\mathbf{D}$, and the continuity equation for the free current density, $\mathbf{J}_{\mathrm{f}}$.

For parts (b) and (c), a steady current is established across the material.

[10 pts] (b) Show that there will be a volume density of free charge in the material given by

$$\rho_{\mathrm{f}}(\mathbf{r}) = -\frac{\nabla\phi}{\sigma} \cdot (\sigma\nabla\epsilon - \epsilon\nabla\sigma),$$

where $\phi(\mathbf{r})$ is the electrostatic potential in the material.

[10 pt] (c) A simple example of an inhomogeneous material is the composition of two different homogeneous materials joined at an interface. Label each region 1 and 2. Show that, at the interface between the two materials, there will be a surface free charge density given by

$$\sigma_{\mathrm{fch}} = J_{\mathrm{n}}\left(\frac{\epsilon_2}{\sigma_2} - \frac{\epsilon_1}{\sigma_1}\right),$$

where $J_n$ is the normal component of the free current density at the interface.

[25 pts] 3. A circular loop in the $xy$ plane is in a spatially uniform magnetic field described by $\mathbf{B} = \hat{\mathbf{z}} B_0 \cos\omega t$. The loop has a radius $a$, resistance $R$, and self-inductance $L$. Find the current in the loop as a function of time.

4. An infinitely long ideal solenoid is at rest in the frame $S'$ with its axis parallel to the $y'$ axis. It has $n'$ turns per unit length and carries a steady current $I'$.

[15 pts] (a) Find $\mathbf{E}$ and $\mathbf{B}$ inside and outside the solinoid for an observer $S$ who traveling at constant velocity $\mathbf{v} = v\hat{\mathbf{x}}$ with respect to $S'$.

[5 pts] (b) Sketch the lines of $\mathbf{E}$ and indicate the charge distribution that must be associated with this electric field.

[5 pts] (c) Will an electric field be observed by someone who sees the solenoid moving at constant speed along the axis of the solenoid? Justify your answer.