E & M

1. (10 pts.) a). Use Maxwell's equations in vacuum and in the absence of charges and currents to show that the electric field $\vec{E}$ and magnetic field $\vec{B}$ satisfy individually wave equations of similar form.

   (10 pts.) b) Assume that an electromagnetic wave with wave vector $\vec{k}$ is described by equations $\vec{E}(\vec{r},t) = \vec{E}_o \sin(\omega t - \vec{k}\cdot\vec{r})$ and $\vec{B}(\vec{r},t) = \vec{B}_o \sin(\omega t - \vec{k}\cdot\vec{r})$, where ω is the wave's angular frequency. Show that $\vec{E}, \vec{B}$ and $\vec{k}$ form an orthogonal triad of vectors.

   (10 pts.) c) Show that the magnitude of $\vec{E}_o$ and $\vec{B}_o$ of the wave are related by a proportionality constant.

   NOTE: $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$, for any vector $\vec{A}$.

2. Consider a long cylinder, surrounded by vacuum, of radius R with volume charge density $\rho = kr$, where r is the distance from its axis and k is a constant.

   (15 pts.) a) Find the electric field vector inside and outside the cylinder.

   (15 pts.) b) Find the electric potential inside and outside the cylinder.

3. Consider a long straight wire of length L that carries a steady current I.

   (15 pts.) a) Find the magnetic vector potential at a perpendicular distance r from the midpoint of the wire.

   (5 pts.) b) Find the magnetic field vector at distances $r \ll L$. Your answer from part (a) may be useful.

   NOTE: $$\int \frac{dx}{\sqrt{a^2+x^2}} = \ell n(x+\sqrt{x^2+a^2})$$

4. A conducting sphere of radius R and conductivity Σ is at time t = 0 uniformly charged with volume charge density ρ_o. Find, as a function of time, the sphere's:

   (10 pts.) a) volume charge density

   (10 pts.) b) surface charge density.