Candidacy Exam **Fall 2012**

Electricity and Magnetism

Instructions: Work any three problems. The total maximum credit is 100 pts but all problems are not worth equal credit.

1. A cylindrical sheet of radius a and infinite length is divided longitudinally into two parts by a plane that contains the axis of the cylinder. The two parts are at constant potentials Φ_0 and $-\Phi_0$, respectively.

   (a) (25 pts) Use symmetry arguments to find the general form of the potential $\Phi(r, \phi)$ as an infinite series at a distance r from the axis, where $r \geq a$.

   (b) (10 pts) Evaluate the expansion coefficients for the series by using the given boundary conditions.

2. The nonrelativistic electric field in the radiation zone for a point charge q with radius vector $\mathbf{r}$ and acceleration $\mathbf{a}$ is

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0 c^2}\frac{1}{r^3}[\mathbf{r} \times (\mathbf{r} \times \mathbf{a})] \ .$$

   (a) (10 pts) Write down the Poynting vector and express it in terms of the electric field.

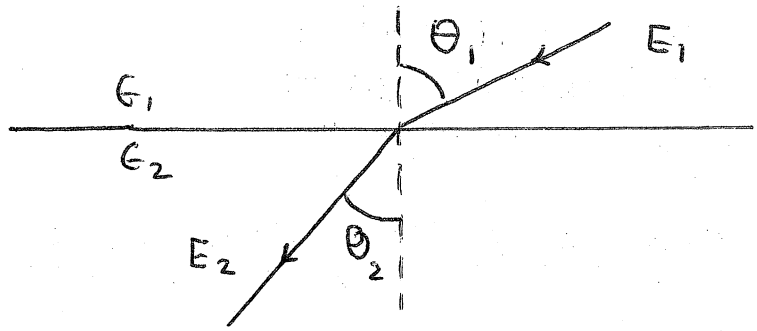
   (b) (15 pts) Calculate the instantaneous power radiated per unit solid angle.

   (c) (10 pts) Derive the Larmor formula for the total instantaneous power.

3. (a) (5 pts) Write the equation for the current density of a moving point charge.

   (b) (25 pts) Now consider a sphere of radius a that carries a uniform surface charge density σ. The sphere is rotated about a diameter with constant angular velocity ω. Use your result from part (a) to calculate the current density $\mathbf{J}$ for the rotating sphere. Take the origin to lie at the center of the sphere and assume that the axis of rotation is along $\hat{z}$.

4. (a) (10 pts) At the interface between a linear dielectric with permittivity ϵ_1 and another with permittivity ϵ_2, the electric field lines bend as shown in the diagram. Find an equation for $\tan\theta_1/\tan\theta_2$ in terms of ϵ_1 and ϵ_2 assuming that there is no *free* charge at the boundary.



(b) (20 pts) Write each of Maxwell's macroscopic equations in differential form and give the common name (if one exists) for the equation. Then use the divergence theorem or Stokes' theorem to rewrite each equation in integral form. Clearly define all quantities such as emf or flux that are relevant.

Possibly useful information:

In cylindrical coordinates,

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2} \ .$$

For $m \neq 0$ and $n \neq 0$,

$$\int_0^{2\pi} \sin n\theta \sin m\theta \ d\theta = \pi\delta_{mn}$$

$$\int_0^{2\pi} \cos n\theta \cos m\theta \ d\theta = \pi\delta_{mn}$$

$$\int_0^{2\pi} \sin n\theta \cos m\theta \ d\theta = 0$$