Candidacy Exam Fall 2010

Electricity and Magnetism

Instructions: Work each of the three problems. The total credit is 100 pts.

1. A dielectric sphere of radius a is centered at the origin. The region of space outside the sphere is air (or vacuum).

   (a) (24 pts) Find the potential $\Phi(r, \theta)$ at any point in space, both inside and outside the sphere, given that the potential on the surface is

   $$\Phi(a, \theta) = A + B \sin^2 \theta \ ,$$

   where A and B are constants. Assume that the potential vanishes at infinity. Here r and θ are standard spherical polar coordinates.

   (b) (10 pts) Determine the sphere's net charge. Under what conditions on A and B does the sphere carry a nonzero net charge?

2. (a) (10 pts) Give an integral equation for the magnetic field, $\mathbf{B}(\mathbf{r})$, produced by an arbitrary current loop that carries current I. Carefully define all symbols in your equation. (Use a diagram.)

   (b) (23 pts) Use your result from part (a) to derive an expression for the magnetic field, $\mathbf{B}(\mathbf{0})$, at the center of a circular current-carrying loop of radius a. The loop lies in the $z = 0$ plane with its center at the origin.

3. (a) (10 pts) State Maxwell's differential equations in macroscopic form (*i.e.*, in a general medium).

   (b) (13 pts) Given the constitutive relations (in SI units) that $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ and $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$, use the inhomogeneous Maxwell equations to determine the density of bound charges $\rho_{\mathrm{b}} (= \rho_{\mathrm{pol}})$ and the corresponding current density $\mathbf{J}_{\mathrm{b}}$ in terms of the polarization $\mathbf{P}$ and magnetization $\mathbf{M}$. (*Hint:* Inhomogeneous equations are those involving source terms.)

   (c) (10 pts) Use your results from part (b) to show that ρ_{b} and $\mathbf{J}_{\mathrm{b}}$ satisfy a continuity equation.

Possibly useful information:

The first few Legendre polynomials are:

$$
\begin{aligned}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_2(x) &= \frac{1}{2}(3x^2 - 1) \\
P_3(x) &= \frac{1}{2}(5x^3 - 3x) \\
P_4(x) &= \frac{1}{8}(35x^4 - 30x^2 + 3)
\end{aligned}
$$