E & M

Answer any 4 of the 5 questions.

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1. Consider the *method of images* for solving certain Poisson's equation problems.
   - (*a*) (5 points) Summarize the general rules for choosing the position and magnitude of image charges in problems involving one flat, grounded, conducting plane.
   - (*b*) (5 points) Describe (but do not attempt to solve!) an example where a single point charge is positioned near two flat, grounded, conducting planes, but the method of images cannot be used.
   - (*c*) (15 points) A single flat, grounded, conductor defines a plane $z = 0$. A charge q is located a height $3h$ above the plane, and another charge $-2q$ is located directly below the first, at a height h above the plane. Use the method of images to find the magnitude and direction of the force on the upper charge.

2. (25 points) According to quantum mechanics, the electron charge density ρ for the ground state hydrogen atom has the form $\rho(r) = -(e/\pi R_B^3)\exp(-2r/R_B)$, where R_B is the Bohr radius. Find the total electric field inside the electron cloud of the atom, at a distance d from the nucleus. Verify that your answer gives the expected limit as d goes to infinity.

3. (25 points) A static magnetic field is given, in Cartesian coordinates, by
$$\mathbf{B} = \frac{B_0}{r_0}(x\,\mathbf{i} - y\,\mathbf{j})$$
where B_0/r_0 is a constant. Consider a region of free space where only this field need be considered; show that Maxwell's equations are satisfied.

4. (25 points) Suppose that the vector and scalar potentials are $\mathbf{A} = A_0\sin(kx - \omega t + \delta)\hat{\mathbf{y}}$ and $V = V_0$, where $A_0,\ k,\ \omega,\ \delta$ and V_0 are constants. Find the electric and magnetic fields. Show that the fields satisfy Maxwell's equations in a vacuum, stating any relationship that must hold among the constants. What can you say about the physical interpretation of the system represented by the above equations for the scalar and vector potentials?

5. A long cylindrical metal shell with inner radius R has a concentric metal wire of radius r_0 inside. A voltage V is applied between the conductors, which are immersed in a non-conducting gas of dielectric constant = 1. The gas breaks down at an electric field E_{max}.
   - (*a*) (5 points) Derive an expression for the electric field between the conductors in terms of their electric charge per unit length.
   - (*b*) (10 points) What choice for r_0 allows V to be as large as possible without breakdown, and what is the corresponding value of V?
   - (*c*) (5 points) What choice for r_0 allows the stored charge per unit length to be as large as possible without breakdown, and what is the corresponding value of V?
   - (*d*) (10 points; includes 5 bonus points) What choice for r_0 allows the stored energy per unit length to be as large as possible without breakdown, and what is the corresponding value of V?