Gray

Candidacy Exam, Fall 2006
Electricity and Magnetism

1. (25 points) Laplace's Equation

   An infinitely long grounded cylinder of radius R is located at the origin so its axis of symmetry is the z-axis. A uniform external field, $E_0\hat{x}$, is applied in the x-direction.

   Solutions to Laplace's equation which are translationally invariant in the z-direction and even functions of the angle, ϕ can be expressed as an expansion,

$$V(\rho,\phi) = \sum_{n=0}^{\infty}\left(A_n\rho^n + \frac{B_n}{\rho^n}\right)\cos(n\phi)\,. \tag{1}$$

   (a) Assume the potential is zero at the origin and hence on the conducting cylinder. The total (net) charge of the cylinder is also zero. Solve for the potential in all space.

   (b) What is the charge density, $\sigma(\phi)$, on the surface of the conducting cylinder?

2. (20 points) Gauss's Law

   A conducting sphere of radius R is covered with an insulating dielectric of thickness R and dielectric permitivity ϵ. The potential of the sphere is V_0. What is the electric field inside the dielectric?

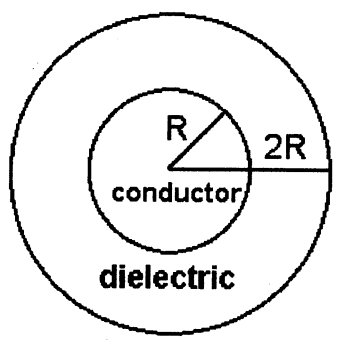

Figure 1: Dielectric covered conducting sphere

3. (30 points) Induction

A long solenoid carries a current that oscillates so that the magnetic field inside the solenoid is essentially uniform and varies sinusoidally in time,

$$\vec{B}(t) = \hat{z}B_0 \sin\left(2\pi\frac{t}{\tau}\right), \tag{2}$$

where τ is a constant.

We will idealize this problem and assume that the time constant, τ, is large compared the time it would take light to travel the length of the solenoid, so there are no radiation effects, just inductive effects. The solenoid is of radius R and its axis of symmetry coincides with the z-axis. The length, L, is much greater than its radius. The thickness of the solenoid is negligible.

(a) According to Faraday's law an electric field is generated inside the solenoid. What is that field?

(b) Outside the solenoid, there is also an electric field is generated. What is that field?

4. Magnetic Field and Potential (25 points)

A circular wire loop of radius R lies in the x-y plane with its center at the origin. The wire carries a current I. (Extensive calculations are not required for the following questions, but some explanation should accompany your answers.)

(a) (10 pts) What is the magnetic field at the origin, $\vec{B}(0)$?

(b) (10 pts) What is the vector potential at the origin, $\vec{A}(0)$?

(c) (5 pts) Far from the origin, the system looks like an ideal magnetic dipole. What is the dipole moment?