E & M

Answer any 4 of the 5 questions.

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1. Starting from either Coulomb's law or Gauss' law, derive an expression for the electric field

   (*a*) (10 points) inside a long solid nonconducting cylinder of radius R, which carries a charge λ per unit length, and

   (*b*) (15 points) at a distance r from an electric dipole of moment $\mathbf{p}$, where the point of interest lies at an angle θ with the direction of $\mathbf{p}$. (You may assume that r is large compared with the separation of the two charges. Remember to express your answer in terms of r, $\mathbf{p}$ and θ.)

2. Assume that the electron can be treated as a charged spherical shell of radius r_0, and further assume that its mass arises from the energy associated with its electric field.

   (*a*) (20 points) Derive an expression for r_0 (known as the classical radius of the electron) in terms of the charge and mass of the electron.

   (*b*) (5 points) Use the values $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg, and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m to verify that the numerical value r_0 comes out comparable to the radius of an atomic nucleus.

3. In most textbooks, Maxwell's equations are first introduced in terms of the field vectors $\mathbf{E}$ and $\mathbf{B}$, and the constants ε_0 and μ_0.

   (*a*) (2 points) Do these equations still hold true in every possible medium? (Yes or no is sufficient).

   (*b*) (2 points) Do these equations still hold true in every possible medium if modified constants ε and μ are substituted? (Yes or no is sufficient).

   (*c*) (7 points) If you answered no to either (a) or (b) above, explain why not. If you answered yes to either (a) or (b) above, explain whether there is any change in interpretation of $\mathbf{E}$, $\mathbf{B}$, or the charge or current densities in the medium compared with in vacuum. (Feel free to reproduce the equations, but you should focus on concepts rather than mathematical detail in your answers).

   (*d*) (7 points) What is the purpose of introducing the vectors $\mathbf{D}$ and $\mathbf{H}$?

   (*e*) (7 points) What is the meaning of *bound current* and *polarization current*?

4. (25 points) A coaxial cable has an inner conductor of radius a and a thin outer shield of radius b. The insulating layer is also very thin, so that b is only slightly larger than a. A current I flows in one direction on the inner conductor, and in the opposite direction on the outer conductor. The current on the inner conductor is distributed uniformly over its cross section. Calculate the energy stored in the magnetic field per unit length of the cable, and the self inductance per unit length of the cable. (You should find that both of these quantities are independent of a and b.)

5. Some MOS-type semiconductor devices can be very susceptible to damage from a small transfer of static charge when handling. Frictional effects can result in a large build-up of excess charge on a person when conditions are unfavorable, but even when this phenomenon can be neglected, an ungrounded person cannot have less than a certain minimum net charge, q_{min}. When a person is grounded, their net charge is affected by thermal fluctuations, and when removed from contact with the ground, the person retains a minimum net charge.

   (*a*) (18 points) Using the famous assumption by physicists that a reasonable model of the human body is an isolated conducting sphere of mass m and density $\rho \sim \rho_{\text{H}_2\text{O}}$, find an approximate expression for $\langle q_{\text{min}}^2 \rangle^{1/2}$ for a person at temperature T. Hint: use equipartition of energy.

   (*b*) (7 points) Substitute plausible values to obtain a rough numerical estimate of $\langle q_{\text{min}}^2 \rangle^{1/2}$. (See question 2 for the numerical values of constants, and assume room temperature, where $kT \sim 25$ meV).