CANDIDACY EXAM: CLASSICAL MECHANICS - SPRING 2013

Answer all problems.

(30 pts) Problem 1: Particle on cylinder

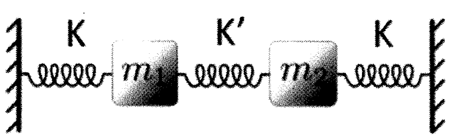
A particle of mass m is constrained to move on a surface of a cylinder defined by $x^2 + y^2 = R^2$. The particle is subject to a force directed toward the origin and proportional to the distance of the particle from the origin: $\vec{F} = -k\vec{r}$. ($r^2 = x^2 + y^2 + z^2$ and k is a positive constant.)

(a) (10p) Calculate the Lagrangian of the particle using appropriate generalized coordinates ($q_i, \dot{q}_i$ for $i = 1, 2, ..N$) .
(b) (6p) Derive the equations of motion in terms of these coordinates.
(c) (4p) Sketch the time dependence of all the coordinates $q_i(t)$ with the given initial conditions: $q_i(t = 0) = 0$ and $\dot{q}_i(t = 0) = \dot{q_{i0}} \neq 0$.
(d) (10p) Find the constraint forces (Q_i) using the method of Langrange's undetermined multipliers.

(40 pts) Problem 2: Coupled oscillations

Consider the linear arrangement of two particles and three springs. Each particle is connected through a spring with spring constant K to a rigid wall and by a spring with spring constant K' to the other particle of same mass m. Consider only longitudinal oscillations.

(a) (20p) Find the eigenfrequencies and normal modes of the system.
(b) (10p) Show that the Langrangian in terms of the normal coordinates is simply a sum of two Lagrangians containing coordinates of each mode separately.
(c) (10p) Find the displacements of the two individual particles ($x_1(t)$, $x_2(t)$) with initial conditions: $x_1(t = 0) = A$, $x_2(t = 0) = 0$ and all particles at rest.



(30 pts) Problem 3: Constants of motion

Consider a particle subject to the following force $\vec{F}(\vec{r}) = \frac{3}{r^5}(\vec{d} \cdot \vec{r})\vec{r} - \frac{\vec{d}}{r^3}$, where $\vec{d}$ is a constant vector. Investigate if the following quantities are constants of motion for a general motion of this particle in this force field. Explicitly demonstrate the required properties of the force field.

(a) (20p) Is the energy conserved? Justify your statement.

(b) (10p) Is the angular momentum conserved? Justify your statement.

(Hints: you might use $\nabla \times (\phi \vec{A}) = \phi(\nabla \times \vec{A}) + \nabla\phi \times \vec{A}$ where ϕ is a scalar and $\vec{A}$ is a vector)