Candidacy Exam Spring 2008

Classical Mechanics

Solve all three problems.

1. **Modified gravity** (40 points)

   Newton's law of gravitation (with the known attractive force $F_G = -GmM/r^2$) allows for closed orbits of two particles like a planet with mass m and sun with mass M. (G is the gravitational constant and r is the distance between the two particles). How would the planetary motion change if the gravitational force had a larger exponent in the r-dependence, making it more attractive for smaller distances? To be specific assume that $F_{new} = -G_{new}mM/r^4$ with a new gravitational constant G_{new}. Answer the following questions.

   (a) Assume the Earth - Sun motion can be described by a circular orbit of radius R_E, under the force F_G. Specify the constants of motion like the total angular momentum L and total energy E as a function of R_E.

   (b) Determine, for the new gravitational force F_{new}, if a stable circular motion is still possible. Explain your answer. (You might consider an effective potential $V_{eff}(r)$ for the radial motion in a rotating system with constant angular momentum.)

   (c) Suppose that Newton's law of gravitation was suddenly changed to F_{new} without affecting the current position and velocity of the planets. Assume that $G_{new} = 3GR_E^2$ to keep the potential unchanged for $r = R_E$. What would happen to the Earth? What would happen to the other planets at a different distance to the Sun? Describe qualitative the motion: does the planet stay on a circular orbit, find a new closed orbit, eventually crash into the sun, escape the solar system for ever or do something else?

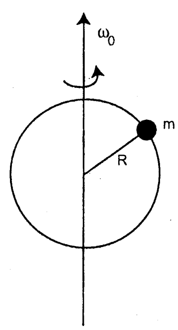
2. **Rotating particle** (30 points)

A particle of mass m is constrained to move on a circle of radius R (without friction). The circle is set to rotate in space with a constant angular velocity ω_0. **The fixed rotation axis is located in the plane of the circle going through its center (see Fig. below).** Do not consider here any gravitational forces.

(a) Find the Lagrangian, generalized momenta and the Hamiltonian and discuss energy conservation.

(b) Find equations of motion for the particle and solve for small displacements from an equilibrium position.

2.



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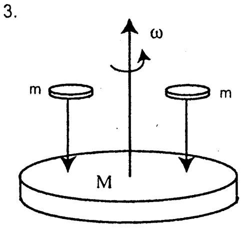

3. **Turntable** (30 points)

A horizontal turntable (a flat cylinder of radius R and mass M) can rotate without friction around the vertical axis in the center (see Fig above). Initially it is brought to a frequency ω_0.

(a) Two coins (with radius r and mass m each) are placed symmetrically along a diameter of the rotating turntable. Assume that the friction between coin and table is high enough that the coins do not move. Explain why the frequency changed. Where would you place the coins to reduce the frequency to $\omega_0/2$ assuming that $r << R$ and $m = M/2$?

(b) Assume the two coins described in (a) are located on the turntable at a distance a from the center. You increase slowly the frequency of the turntable until the coins start moving at ω_{max}. Explain why and how the coins start moving. Derive an expression to evaluate the coefficient of static friction μ_S between the coin and turntable from this experiment.