SPRING 2005

CLASSICAL MECHANICS

1. Derive Kepler's Third Law of Planetary Motion, $P^2 = C_1 a^3$, that the square of the period for a planet of mass *m* is directly proportional to the cube of its average distance from the Sun, which you should take to have mass *M*. (Do not assume that $m << M$, but do assume circular orbits.) Obtain an expression for C_1 .

2. A body of uniform cross-sectional area $A = 1\ cm^2$ and of mass density $\rho = 0.8\ g/cm^3$ floats in a liquid of density $\rho_0 = 1\ g/cm^3$ and at equilibrium displaces a volume $V = 0.8\ cm^3$. Show that the period of small oscillations about the equilibrium position is given by

$$\tau = 2\pi \sqrt{V/gA} \ ,$$

where g is the gravitational field strength. Determine the value of τ . Ignore viscosity. (Hint: At equilibrium, the buoyant force equals the weight of the body.)

3. A sphere of radius ρ and mass m is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of inside radius R. Using the angles θ and φ as shown in the figure as the generalized coordinates, determine the Lagrangian function, the equation of constraint, and Lagrange's equations of motion. Find the frequency of small oscillations. (The moment of inertia of a sphere about any diameter is, $I = (2/5)\ m\ \rho^2$.)