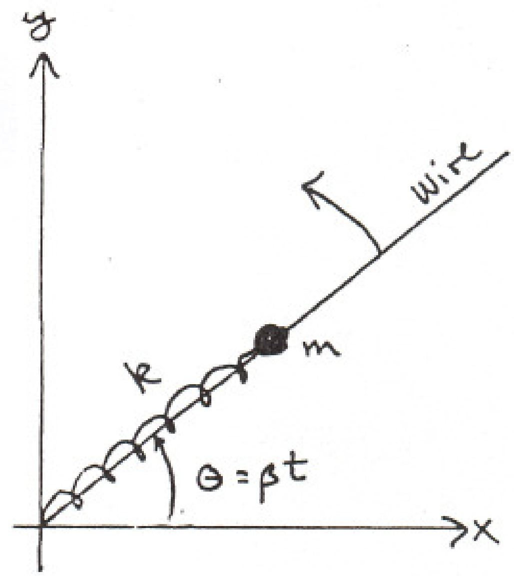
Classical Mechanics

1. (25 pts.) A pi meson (π) of mass m_π at rest disintegrates into a muon (μ) of mass m_μ and a massless neutrino (ν). Show that the kinetic energy (T_μ) of the muon is

$$T_\mu = \frac{(m_\pi - m_\mu)^2 c^2}{2 m_\pi}.$$

2. A particle of mass m slides along a smooth, straight wire in the x-y plane. Wrapped around the wire, and attached to the particle is a spring of force constant k and unstretched length r_0. Both the wire and the spring are attached to the origin. The wire rotates with $\theta = \beta t$, where β is a constant.



($\vec{v} = r \cdot \vec{e}_r + r \cdot \theta \cdot \vec{e}_\theta$ in polar coordinates.)

(10 pts.) a. Construct the Lagrangian for this system.

(10 pts.) b. Solve Lagrange's equation for this system to find the equation of motion.

(5 pts.) c. If $\beta^2 < k/m$, how would you characterize the radial motion?

3. (5 pts.) a. State Hamilton's Principle in its integral form for conservative systems.

(20 pts.) b. A body is released from a height of 64 ft and 2 sec later it strikes the ground. The equation for the distance of fall s during a time t could conceivably have any of the forms (where g has different units in the three expressions)

$$s = gt; \qquad s = (1/2)\, gt^2; \quad s = (1/4) gt^3$$

all of which yield s = 64 ft for t = 2 sec. Show that the integral in Hamilton's Principle is smallest for the correct one of these three forms, by calculating the integral explicitly for this time integral. You may assume V = -mgs.

4. (25 pts.) A particle of mass m, moving at relativistic speed v experiences a force given by

$\vec{F} = -\nabla V(\vec{r})$. The Lagrangian that gives the correct equations of motion is

$$\mathcal{L} = mc^2 \left(1 - \sqrt{1 - v^2/c^2}\right) - V(\vec{r}).$$

Use the definition of the Hamiltonian in terms of the Lagrangian to determine the relativistic kinetic energy, T, in terms of m, v and c.