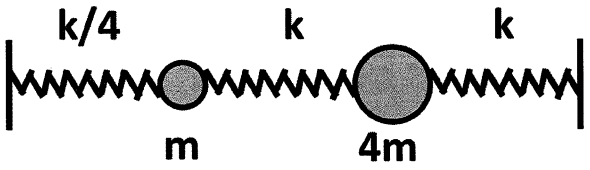
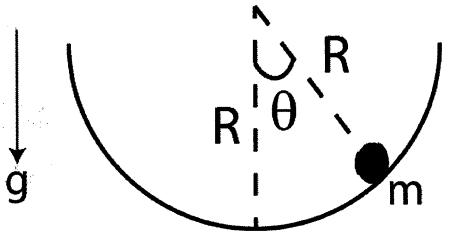
Physics Candidacy Exam, Fall 2012, Classical Mechanics

Question 1: (30 points) Two particles of mass m and 4m are connected by springs of stiffness k/4, k, and k as shown in the figure below. Find the eigenfrequencies of normal modes.



Question 2: (30 points, 6 points each section) A point particle of mass m slides without friction on the inside of a hemispherical bowl of radius R. Describe the location of the particle in terms of polar and azimuthal angles θ and ϕ and assume gravitational field points along the axis of sphere.



a) Write the Lagrangian for the motion.

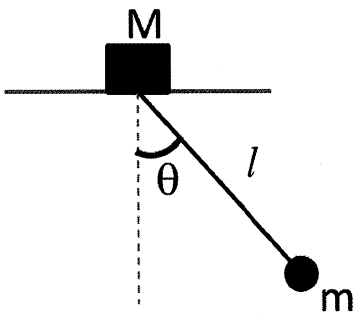
b) Determine formulas for the generalized momenta p_θ and p_ϕ

c) Write the Hamiltonian for the motion

d) Develop Hamilton's equations of motion

e) If at t=0, $\theta=\theta_0$ and $\dot{\theta} = 0$ and $\dot{\phi} = 0$, calculate the maximum speed at a later time.

Question 3 (30 points, 15 points each section): A pendulum of length l and mass m is suspended from a block of mass M. The block moves without friction and is constrained to move in the horizontal direction (x-axis). Assume all motion of m is confined to the xy plane. At t=0 both masses are at rest, the horizontal coordinate of the block is x_0, and the pendulum has angular deflection $\theta=\theta_0$ with respect to y-axis. Consider masses to be point particles.



a) Write down the Lagrangian for this system.

b) Find the equation of motion assuming $\theta, \dot{\theta}$, and the speed of M to be all $\ll 1$. Keep terms up to quadratic powers of these variables.

Question 4 (10 points): A satellite is in circular orbit at a distance R_0 above the center of the earth. The satellite experiences a viscous drag force of magnitude $F_v = Av^{\alpha}$, where v is the velocity of the satellite. This drag force causes a change in the radial distance of the satellite given by $dr/_{dt} = -C$, where C is a positive constant, and a change in energy given by $dE/_{dt} = -F_v v$. Obtain expressions for A and α assuming the loss of energy per orbit is small compared to the total kinetic energy. Take mass of earth to be M and mass of satellite to be m.