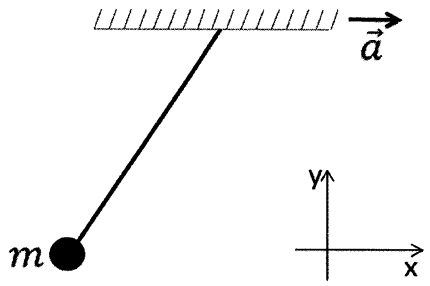
Fall 2010

Classical Mechanics Questions

1- (50 points) A simple pendulum of length l and a bob of mass m is attached to a platform, of negligible mass, that has a constant acceleration $\vec{a}$ in the x-direction. The pendulum can oscillate only in the xy-plane. Assume the pivot point of the pendulum is at $x = 0$ and has an initial velocity of $\dot{x} = v_0$ at $t = 0$.



a) (10 points) What are the kinetic energy, potential energy, and Lagrangian for the pendulum? Use appropriate generalized coordinate(s).

b) (10 points) Find the Lagrange equation(s) of motion.

c) (6 points) Find the equilibrium angle θ_e.

d) (10 points) Find the frequency of small oscillations about the equilibrium angle θ_e.

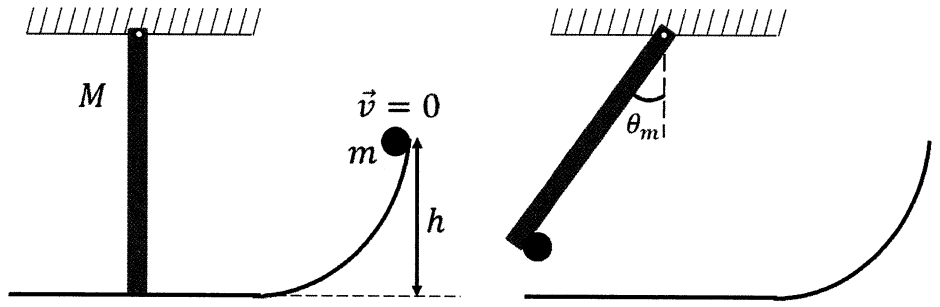
e) (8 points) How would you derive the Hamiltonian for this system? You do not need to actually derive it, but just describe the steps to drive it.

f) (6 points) Given that the Hamilton is

$$H = \frac{{p_\theta}^2}{2ml^2} - \frac{(v_0+at)p_\theta cos\theta}{l} - \frac{1}{2}m(v_0 + at)^2 \sin^2\theta - mglcos\theta,$$

find the Hamilton's equations of motion? Is the mechanical energy conserved, and why?

2- (20 points) In this problem we will consider a point particle of mass m colliding with a rod of length l and mass M pivoted at one end as shown in the figure below. The particle slides on a frictionless surface from a height h, and hits the rod at its edge. The particle sticks to the rod, and the rod-particle system rotates about the pivot point as shown in the figure. **Find the maximum angle of rotation θ_m.** The moment of inertia of this rod about an axis that is perpendicular to the rod and passes through the center of mass of the rod is $I_{COM} = \frac{1}{12}Ml^2$.



3- (30 points) Consider an infinitely long continuous string with tension τ. A bead of mass m is attached to the string at x=0. A wave train with velocity ω/k is incident from the left, and a reflection and transmission occur at x=0. Find the reflection and transmission coefficients for this system.