Candidacy Exam **Fall 2009**

Classical Mechanics

Solve all four problems.

1. **Pendulum** (30 points)

Consider a pendulum composed of a point mass m suspended from a weightless, rigid arm of length L. Initially the mass is at rest at an angle ϕ_0 from the vertical.

(a) Derive the Lagrangian, Lagrange's equations of motion, and solve for $\phi(t)$ for small initial angles ϕ_0 .

(b) Use Lagrange's equations including Lagrange undetermined multiplier(s) to get an expression for the forces of contraint (in differential form) in the radial (F_r) and tangential (F_ϕ) direction for general ϕ_0.

(c) Find the time dependence for both forces for small ϕ_0 and sketch the motion $\phi(t)$ and the forces $F_r(t)$ and $F_\phi(t)$.

2. **Kepler's law** (15 points)

Kepler's 2nd law for planetary motion states that the area swept by the radius vector (the distance between the two particles) per time is constant: $dA/dt = const.$

(a) Derive this law: $dA/dt = const.$

(b) For what forces does this law hold? Does a harmonic force ($F = -kr$) give the same results as the gravitational force?

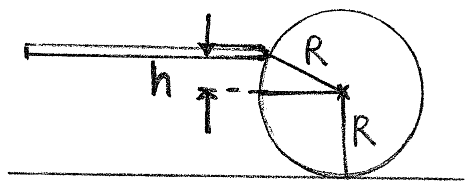
3. **Billard cylinder** (25 points)

A cylinder on a horizontal table is subject to an impulse force applied over a small area of the cylinder (in the middle of its length). In order for the cylinder to roll without slipping on a horizontal table, the impulse must be applied horizontally at a certain height h above the center of the cylinder (see figure). In the following, always assume a very high friction table and cylinder surface.

(a) Show that the moment of inertia I_{cyl} for a solid homogeneous cylinder of mass M and radius R for rotation around the cylinder axis is $I_{cyl} = \frac{1}{2}MR^2$

(b) What is the right height h to strike a solid homogeneous cylinder with radius R and length l to make it roll around the cylinder axis on the table without slipping?

(c) Could you strike a hollow (very thin walled) cylinder to make it roll without slipping? Explain your answer.



4. **Drude model** (30 points)

Consider the one-dimensional motion of a particle in a time dependent force field with dissipation: The simplest model to describe the electric conductivity of electrons in a metal leading to Ohm's law is the Drude model, where the electron of mass m is subjected to an electric field with force $F = -eE$ and the scattering with phonons (characterized by the time between collisions τ) is approximated by a velocity dependent force $F_r = -mv/\tau$.

(a) Derive the terminal velocity v_t of the electron in a constant electric field $E(t) = E_0 = const.$

(b) Derive the velocity of the electron $v(t)$ in an alternating field $E(t) = E_0 \cos(\omega t)$

(c) Sketch the amplitudes of $v(t)$ as function of external frequency ω:
Separate the amplitudes into in-phase amplitude ($\Delta\phi = 0$) and out-of-phase amplitude ($\Delta\phi = \pi/2$), with phase shift $\Delta\phi$ between v and E. Determine at which ω the amplitudes show a maximum.