Classical Mechanics

Candidacy Exam Fall 2008

1. 35 pts. A particle's motion in 3 dimensions is described by the Lagrangian $L(\vec{r}(t), \dot{\vec{r}}(t))$, and the system is conservative.

   a. If L is invariant to an infinitesimal translation of the origin of the coordinate system along direction $\hat{n}$ show that this implies conservation of linear momentum in direction $\hat{n}$.

   b. If L is invariant to an infinitesimal rotation of the coordinate system about $\hat{n}$ show that this implies conservation of angular momentum in direction $\hat{n}$.

   c. L is <u>not</u> invariant to an infinitesimal translation of the time parameter. Show that the Hamiltonian function, derived from L via the Legendre transformation, is invariant to time translation.

2. 33 pts. A particle of mass m is constrained to frictionless gravitational motion along the inside surface of a cone whose axis is vertical, its apex points down, and its half angle at the apex is α.

   a. Identify the cyclic coordinate and the associated conserved generalized momentum p_c.

   b. A circular orbit is possible. Find its radius in terms of p_c.

   c. With p_c fixed by the initial conditions, find out whether the circular orbit is stable or not, to small perturbations.

3. 32 pts. Find the resonant frequencies for small oscillations of the double pendulum in which a string of length ℓ suspends a mass m from which a second mass αm is suspended by a string of length $\beta\ell$. Take the mass ratio α =2 and the string length ratio β = ½.