CANDIDACY EXAM AUGUST 2005

Classical Mechanics

For full credit, answer ALL questions. Each question carries equal credit. Labeled subparts within questions carry equal credit.

---

1. A meteorite (mass m) falls vertically to the earth (mass M). Assume that the meteorite started a long distance r_0 away ($r_0 \gg$ the earth's radius), with zero relative speed. Use Lagrange's equation to find the meteorite's speed as a function of distance from the center of the earth. Consider only distances beyond the earth's atmosphere.

2. An unstable particle with mean proper lifetime τ decays in flight. There are N decay products, having rest masses m_1 .... m_N and laboratory momentum components (p_{1X}, p_{1Y}, p_{1Z}) .... (p_{NX}, p_{NY}, p_{NZ}). In terms of some or all of the above quantities, write down relativistically correct expressions for
(a) the total energy E of the system in the laboratory frame,
(b) the mass m_0 of the original particle,
(c) the probability that the particle lives for a time t_0 *or longer* in the laboratory frame, and
(d) the kinetic energy release of the decay in its rest frame.
Note: you may use the symbols E and/or m_0 in your answer for parts (c) and (d).

3. A planet of mass m orbits a star of mass M. A spherical distribution of dust with uniform density ρ surrounds the star, and extends past the planet's orbit. You may assume that the mean radius of the orbit, R, is very large compared with the radius of the star. Ignore the drag force due to the dust.
(a) What is the central force when the planet is at a distance r from the star?
(b) Next, consider a circular orbit with angular momentum ℓ. Find the equation connecting R and the other parameters given above. It is sufficient to find a polynomial expression in R (do not try to solve for R).
(c) Finally, consider small deviations from a circular orbit in the approximation of low dust density. Discuss qualitatively the type of orbit that would occur.

4. Two identical springs, of force constant k, are fastened end-to-end. One end of the double spring is fixed to the ceiling, and a mass m hangs down vertically from the far end. A mass $2m$ is fastened to the middle, where the two springs join. The whole system moves in the vertical direction only, without friction. Find the eigenfrequencies for small oscillations.