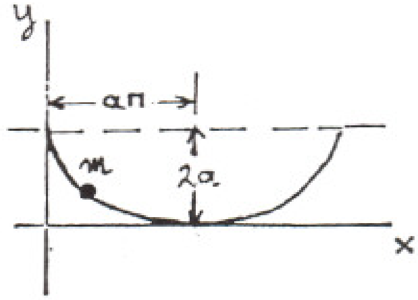
Candidacy Exam
Fall 2004

CLASSICAL MECHANICS

Instructions: Solve all four problems.

1. (20 points) If the Hamiltonian function H of a conservative system is expressed as a function of the coordinates q_i and momenta p_i and does not contain t (time) explicitly, prove:
(a) Hamilton's canonical equations of motion,
(b) that H is a constant, and
(c) that H is equal to the total energy of the system.
Assume that the kinetic energy of the system is a homogeneous quadratic function of the $\dot{q}_i$'s, and that the potential energy is a function of the q_i's only.

2. (30 points) A bead of mass m slides without friction on a wire in the shape of a cycloid with equations:
$x = a(\theta - \sin\theta), \qquad y = a(1 + \cos\theta),$
where $0 \leq \theta \leq 2\pi$.

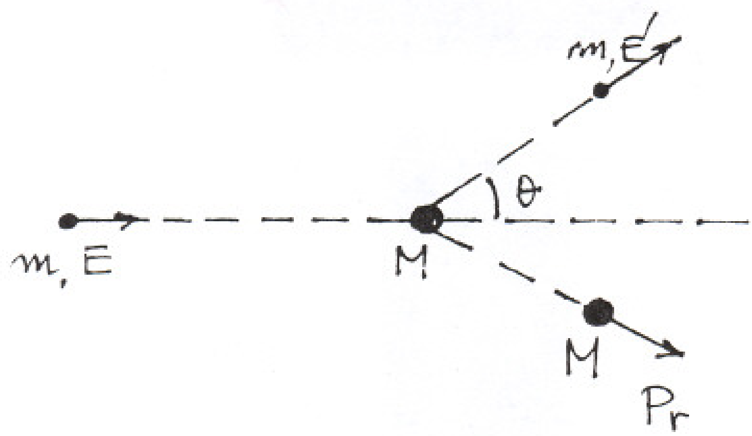


(a) Find the Lagrangian function and
(b) the equation of motion of the bead.
(c) Show that the equation of motion found in question (b) can be written

$$\frac{d^2u}{dt^2} + \frac{g}{4a}u = 0$$

where $u = \cos(\theta/2)$.
(d) Show that the bead oscillates and find its period.

3. (25 points) A <u>relativistic</u> electron of mass m and energy E ($m \ll E$) collides elastically with a stationary proton of mass M. The electron is scattered at an angle θ relative to its initial direction. Find the energy E' of the scattered electron in terms of E, θ, and M.



4. (25 points) A flexible, inextensible string of length l and linear mass density λ is positioned on a table as shown in the figure. At $t = 0$ a length y_o of the string is hanging down vertically from the table. The string is released so that at any subsequent time later it is moving with a speed $v = dy/dt$, where y is the length of string that is hanging. Find:

(a) the position y of the string as a function of time,
(b) the speed v of the string as a function of time,
(c) the speed of the string the moment it leaves the table.
(Assume no friction or air resistance present)