Candidacy Exam **Fall 2002**

Classical Mechanics

Instructions: Work each of the following three problems.

1. A particle of mass m is described by the Hamiltonian,

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \omega \cdot \mathbf{L} + V(\mathbf{r}) \ ,$$

where ω is a constant vector and $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ so that we may think of the term $-\omega \cdot \mathbf{L}$ as spin-orbit potential.

(a) (12 pts) Determine the Hamilton equations of motion. *Hint:* The vector identity $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$ may be useful.)

(b) (12 pts) Eliminate the canonical momentum $\mathbf{p}$ from Hamilton's equations to find the $\mathbf{r}$ equation of motion and show that it corresponds to a particle moving in a rotating reference frame. Identify the Coriolis and centrifugal force terms.

(c) (12 pts) Is the Hamiltonian conserved? Why or why not? Determine the Lagrangian $\mathcal{L} = \mathcal{L}(\mathbf{r}, \dot{\mathbf{r}}, t)$.

2. (30 pts) A particle of mass m and angular momentum L moves under the influence of a central force $f(r)$. Given that its orbit equation is $r = 2a\cos\theta$, where a is a constant, determine $f(r)$.

3. (34 pts) A linear triatomic molecule can be described as two equal masses m interacting with a third mass M through forces that can be represented as due to springs, each of unstretched length b and spring constant k, as shown below. Determine the normal frequencies for small longitudinal oscillations of the system.