Candidacy Exam Fall 2000

Classical Mechanics

Do all problems

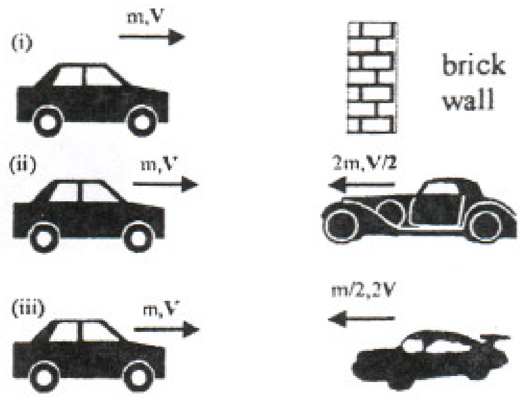
(1) (30 pt., divided equally between A and B)

(A)Consider three totally inelastic collisions (i.e., the two colliding objects are intertwined after the collision), pictured at right moments before the collision.

(a) After which collision(s) is the left-hand car totally stationary?

(b) Which collision(s) results in the maximum damage to the car(s)?

Explain your reasoning in both cases.



(B) Consider two symmetric cylinders, of the same length, radius, and mass. They are coated to appear identical. However, one is solid while the other is hollow, with a high-density outer shell and thin end caps. Describe and explain a non-destructive test to determine which cylinder is hollow.

(2) (40 pts) Consider the interparticle potential $U(r)=\varepsilon\left(\left(\frac{\sigma}{r}\right)^{12}-2\left(\frac{\sigma}{r}\right)^{6}\right)$, where σ and ε are constants characterizing the length and strength of the potential. This "6-12" or Lennard-Jones potential has often been used to model interparticle interactions, in a liquid for example.

(a) For an appropriate set of generalized coordinates, find the Lagrangian, the associated generalized momenta, and the Hamiltonian for two particles of mass m interacting via this potential in 2-dimensional space. Remember that: (a) The kinetic energy can be rewritten as the sum of the kinetic energy associated with the center of mass plus a term involving the reduced mass μ and the vector **r** between the two particles. (b) Conservation of angular momentum means that a three-dimensional system will be confined to a plane determined by the initial conditions, so that the two-dimensional case can be directly extended to the three-dimensional one.

(b) Find the equations of motion for the system.

(c) List and justify all conserved quantities for this system.

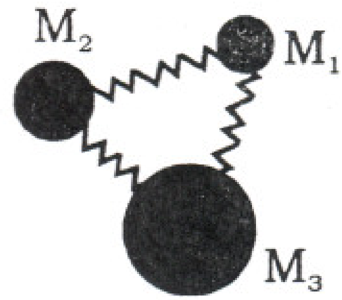
(d) Using the conservation laws, restate the problem in terms of a one-dimensional one with an effective potential $V_{eff}(r)=U(r)+\ell^2/2\mu r^2$, where ℓ is the angular momentum and μ=m/2 is the reduced mass.

(e) Sketch U(r), being careful to show right asymptotic behavior and the position of any minima or maxima. On the same graph, roughly sketch $V_{eff}(r)$ for two different values of ℓ:

$\frac{\ell^2}{2\mu}\sim\frac{\varepsilon\sigma^2}{2}$ and $\frac{\ell^2}{2\mu} >> \varepsilon\sigma^2$ where μ=m/2 is the reduced mass.

(c) Using this sketch, describe qualitatively the different types of motion you might see in this system, indicating the different energy ranges on your sketch.

(3) (30 pts) **note: you do not need any thermodynamics, beyond that stated here, to solve this problem.** Consider an elastically bound triatomic molecule as sketched. Suppose that a crowd of n such molecules per unit volume is in thermal equilibrium at temperature T. In the classical limit, the equipartition theorem states that, on average, there is a kinetic energy $\frac{1}{2}k_BT$ associated with each degree of freedom and an energy $\frac{1}{2}k_BT$ associated with the potential energy of each normal mode of oscillation of the molecule. Assume the molecules are independent (i.e. ideal gas).



(a) How many degrees of freedom are there? How many normal modes of oscillation are there?

(b) Write down and account for the various contributions to the total thermal energy E(T) possessed by each molecule on average.

(c) The rate at which any molecular property α is transferred across a unit area in a gas, J_α, can be written in terms of the molecular bombardment rate J_n per unit area, the mean free path $\lambda = 1/n\sigma$ and the molecular mass $M = M_1 + M_2 + M_3$. Thus,

$$J_\alpha = -\frac{d\alpha}{dx} J_n \lambda = -\frac{d\alpha}{dx}\frac{1}{\sigma}\sqrt{\frac{k_B T}{4M}} \qquad [1]$$

Now the property α can perfectly well be the total thermal energy E(T) possessed by the molecules, as calculated in part (b). Furthermore, we know

$$\left(\frac{d\alpha}{dx} \equiv\right)\frac{dE}{dx} = \frac{dE}{dT}\frac{dT}{dx} \qquad [2]$$

Use eqs. 1 and 2 with the results of part (b) to calculate the thermal conductivity κ of the triatomic gas. (hint: κ is expressible in units of $\left(\frac{Joules}{m^2}\right)\left(\frac{1}{s}\right)\left(\frac{m}{\deg K}\right)$.)