

Masses and widths of N and Δ resonances

D. M. Manley

Department of Physics and Center for Nuclear Research, Kent State University, Kent, Ohio 44242

(Received 5 July 1994)

A simple approximation is derived to relate the complex pole in a resonant partial-wave T -matrix amplitude to the conventional Breit-Wigner parameters that describe the scattering resonance. The approximation is tested by using well-established resonance parameters for the $\Delta(1232)_{\frac{3}{2}^+}$. This new result facilitates a comparison of both Breit-Wigner parameters and pole positions for 17 resonances described by four major analyses of πN scattering.

PACS number(s): 13.75.Gx, 14.20.Gk

I. INTRODUCTION

Resonance parameters are generally determined by analyzing the energy dependence of experimentally determined partial-wave amplitudes that describe resonant two-body scattering reactions. Typically the amplitudes are fitted with generalized Breit-Wigner formulas that contain a "mass parameter" M and a "width parameter" Γ . While the conventional Breit-Wigner parameters M and Γ are more often compared with model predictions, the pole positions are considered more fundamental and less model dependent [1-4]. This paper provides a simple approximate way to relate M and Γ to the pole position. In addition, questions concerning properties of specific resonances, such as the width of the Roper resonance, are addressed.

In the vicinity of a pole, a partial-wave T -matrix amplitude can be approximated by the Breit-Wigner form

$$T \approx \frac{f(W)}{M - W - i\Gamma(W)/2}, \quad (1)$$

where W is the total invariant energy in the center-of-mass (c.m.) frame. The energy dependence of $\Gamma(W)$ displaces the real part of the pole position from M , and the imaginary part is different from $-i\Gamma/2$, where Γ denotes $\Gamma(W)$ evaluated at $W = M$. If $W_p = M_0 - i\Gamma_0/2$ denotes the complex pole, then $D(W_p) = 0$ where $D(W) = M - W - i\Gamma(W)/2$. Newton's method for determining the root of a function indicates that

$$W_p \approx W_0 - \frac{D(W_0)}{D'(W_0)}, \quad (2)$$

where $D'(W) = dD(W)/dW$ and $W_0 = M - i\Gamma/2$ is an initial approximation for the pole position [5]. For $M \gg \Gamma/2$, I can make the Taylor series expansions $\Gamma(W_0) \approx \Gamma - i\alpha\Gamma$ and $\Gamma'(W_0) \approx \Gamma'$, where Γ' denotes $d\Gamma(W)/dW$ evaluated at $W = M$ and $\alpha = \Gamma'/2$. Upon substituting into Eq. (2) and simplifying, I obtain

$$M_0 \approx M - \frac{\Gamma}{2} \left(\frac{\alpha}{1 + \alpha^2} \right), \quad (3a)$$

$$\Gamma_0 \approx \frac{\Gamma}{1 + \alpha^2}. \quad (3b)$$

This approximation is somewhat similar to that derived by Lichtenberg [4], who found by a Taylor-series expansion of $D(W)$ about the point $W = M$ that (in my notation) $M_0 \approx M - (\Gamma/2)\alpha$ and $\Gamma_0 \approx \Gamma [1 - \alpha^2 - (\Gamma/4)\alpha']$, with $\alpha' = \Gamma''/2$. Clearly Eq. (3) reduces to Lichtenberg's result if $\alpha^2 \ll 1$ and if $|\alpha'| \ll (4/\Gamma)\alpha^2$. Both conditions are approximately satisfied for the $\Delta(1232)_{\frac{3}{2}^+}$, as discussed in the following section.

The approximations above are not directly useful for determining the residues at the pole positions, which clearly require knowledge of the explicit form of the function $f(W)$ in Eq. (1). This function is generally very model dependent and may be affected strongly by background contributions (from nonresonant terms and/or from overlapping resonances). An advantage of the approximations given in Eq. (3) is that they provide a convenient way to investigate how values of the conventional Breit-Wigner parameters will vary depending on the assumed energy dependence of $\Gamma(W)$. [Values of the pole parameters are thought to be approximately insensitive to the assumed energy dependence of $D(W)$].

II. DISCUSSION

As a specific example and a test of Eq. (3), consider the $\Delta(1232)_{\frac{3}{2}^+}$, the first resonance in elastic pion-nucleon scattering. If I write

$$\Gamma(W) = \Gamma \frac{\rho(W)}{\rho(M)}, \quad (4)$$

where $\rho(W)$ is a phase-space factor, then it follows that

$$\alpha = \frac{\Gamma}{2} \frac{\rho'(M)}{\rho(M)}. \quad (5)$$

For the $\Delta(1232)_{\frac{3}{2}^+}$, the phase-space factor may be parametrized as [6]

$$\rho(W) = \frac{q}{W} B_\ell^2(qR), \quad (6)$$

where B_ℓ is a Blatt-Weisskopf barrier-penetration fac-

tor [7], $\ell = 1$ is the orbital angular momentum between the pion and nucleon, and q is the relative momentum between the pair in the c.m. frame; i.e., $W = \sqrt{q^2 + m_\pi^2} + \sqrt{q^2 + m_N^2}$, where m_π and m_N are the masses of the pion and nucleon, respectively. For the $\Delta(1232)$, I take $M = 1231$ MeV and $\Gamma = 118$ MeV [6]. Then using $m_\pi = 140$ MeV and $m_N = 938$ MeV, I find $q = 226$ MeV at $W = M$. Using $B_1(x) = x/\sqrt{1+x^2}$ [8] and the value $R = 1.0$ fm from Ref. [6], it is straightforward to calculate that $\alpha = 0.40$. With this value substituted into Eq. (3), I find $M_0 = 1211$ MeV and $\Gamma_0 = 102$ MeV, in excellent agreement with accepted values for the pole position [9,10]. By comparison, if Lichtenberg's approximation [4] is used, then the values obtained are $M_0 = 1207$ MeV and $\Gamma_0 = 102$ MeV, where, for the width, I have used the calculated value, $\alpha' = -8.0 \times 10^{-4}$ MeV $^{-1}$. As Lichtenberg has noted, his approximation effectively provides a lower bound for the mass and width.

From Eq. (3), it follows that values of M , M_0 , and Γ_0 can be used to determine values of α according to the expression, $\alpha = 2(M - M_0)/\Gamma_0$. Table I lists calculated values of α for several N and Δ resonances observed in πN elastic scattering. These values were determined using the pole positions and mass parameters from the partial-wave analysis of Cutkosky *et al.* [9]. Note that in each case, $0 < \alpha < 1$, which is reflected in the observation that $M > M_0$ and $\Gamma > \Gamma_0$ [4]. This result is easily understood from Eq. (6), which implies that $\rho(W)$ is a monotonically increasing function for real values of W [$\rho(W) \propto q^{2\ell+1}$ for small q and $\rho(W) \rightarrow \text{const}$ for large q]. I next calculated the corresponding width parameters according to Eq. (3), viz., $\Gamma_{\text{calc}} = (1 + \alpha^2)\Gamma_0$. For each resonance in

Table I, the difference between Γ_{calc} and the value of Γ determined by Cutkosky *et al.* is generally much smaller than the quoted uncertainty in Γ . [Note that Table I lists $\alpha = 0.44$ for the $\Delta(1232)_{\frac{3}{2}^+}$. This value corresponds to an interaction radius of $R = 0.85$ fm in Eq. (6).]

Conventional Breit-Wigner parameters M and Γ were then calculated from the pole positions (solution SM90) determined in a recent partial-wave analysis of elastic πN scattering by Arndt *et al.* [10]. (An even more recent set of partial-wave analyses employing rigorous constraints from simultaneous forward and fixed- t dispersion relations has been performed, although new pole positions are not available [11].) Results of the calculations, which used Eq. (3) with values of α from Table I, are listed in Tables II and III under the heading VPI91. Tables II and III compare these parameters with those published for three other analyses: namely, the 1992 Kent State University analysis (KSU92) [6], the 1980 Carnegie-Mellon-Berkeley analysis (CMB80) [9], and the 1980 Karlsruhe-Helsinki analysis (KH80) [12]. The tables also give unweighted means with sample standard deviations. In most cases, the different analyses agree surprisingly well for the parameter values; however, there are six cases that warrant further comment.

Three analyses of resonance formation in πN scattering experiments indicate that the Roper resonance [$N(1440)_{\frac{1}{2}^+}$] is broad, in disagreement with the KH80 result (see Table II). It should be noted that the entry in Table II under the column VPI91 was determined from the P_{11} pole at $(1360 - i 126)$ MeV; this pole is on the sheet of the Riemann surface most directly reached by analytic continuation from the real axis. An auxil-

TABLE I. Comparison of the calculated width parameters Γ_{calc} with values of Γ from the πN partial-wave analysis of Cutkosky *et al.* [9]. All masses and widths are in MeV. The values of α were determined as discussed in the text. Note that $0 < \alpha < 1$ for all cases.

Resonance	M_0	Γ_0	M	Γ	Γ_{calc}	α
$N(1440)_{\frac{1}{2}^+}$	1375 ± 30	180 ± 40	1440 ± 30	340 ± 70	273	0.72
$N(1520)_{\frac{3}{2}^-}$	1510 ± 5	114 ± 10	1525 ± 10	120 ± 15	122	0.26
$N(1535)_{\frac{1}{2}^-}$	1510 ± 50	260 ± 80	1550 ± 40	240 ± 80	285	0.31
$N(1650)_{\frac{1}{2}^-}$	1640 ± 20	150 ± 30	1650 ± 30	150 ± 40	153	0.13
$N(1675)_{\frac{5}{2}^-}$	1660 ± 10	140 ± 10	1675 ± 10	160 ± 20	146	0.21
$N(1680)_{\frac{5}{2}^+}$	1667 ± 5	110 ± 10	1680 ± 10	120 ± 10	116	0.24
$N(1710)_{\frac{1}{2}^+}$	1690 ± 20	80 ± 20	1700 ± 50	90 ± 30	85	0.25
$N(1720)_{\frac{3}{2}^+}$	1680 ± 30	120 ± 40	1700 ± 50	125 ± 70	133	0.33
$N(2190)_{\frac{7}{2}^-}$	2100 ± 50	400 ± 160	2200 ± 70	500 ± 150	500	0.50
$N(2220)_{\frac{9}{2}^+}$	2160 ± 80	480 ± 100	2230 ± 80	500 ± 150	521	0.29
$N(2250)_{\frac{9}{2}^-}$	2150 ± 50	360 ± 100	2250 ± 80	480 ± 120	471	0.56
$\Delta(1232)_{\frac{3}{2}^+}$	1210 ± 1	100 ± 2	1232 ± 3	120 ± 5	119	0.44
$\Delta(1600)_{\frac{3}{2}^+}$	1550 ± 40	200 ± 60	1600 ± 50	300 ± 100	250	0.50
$\Delta(1620)_{\frac{1}{2}^-}$	1600 ± 15	120 ± 20	1620 ± 20	140 ± 20	133	0.33
$\Delta(1700)_{\frac{3}{2}^-}$	1675 ± 25	220 ± 40	1710 ± 30	280 ± 80	242	0.32
$\Delta(1905)_{\frac{5}{2}^+}$	1830 ± 40	280 ± 60	1910 ± 30	400 ± 100	371	0.57
$\Delta(1910)_{\frac{1}{2}^+}$	1880 ± 30	200 ± 40	1910 ± 40	225 ± 50	218	0.30
$\Delta(1930)_{\frac{5}{2}^-}$	1890 ± 50	260 ± 60	1940 ± 30	320 ± 60	298	0.38
$\Delta(1950)_{\frac{7}{2}^+}$	1890 ± 15	260 ± 40	1950 ± 15	340 ± 50	315	0.46

iary pole, at $(1413 - i 128)$ MeV, was found on another sheet, and has less physical significance for the multichannel Roper resonance. Further discussion of the issue of poles in the P_{11} partial-wave amplitudes can be found in the Brief Report by Cutkosky and Wang [13]. Recently Höhler determined pole positions based on a speed-plot analysis of the KH80 amplitudes [14]. For the Roper resonance, he found $\Gamma_0 = 164$ MeV, which suggests, based on arguments presented earlier, that the KH80 width parameter ($\Gamma = 135$ MeV) is too small for this resonance. Using the mean value $M = 1441$ MeV from Table II and mean pole values (see below) of $M_0 = 1372$ MeV and $\Gamma_0 = 213$ MeV, the calculated width parameter is $\Gamma = 300$ MeV, which corresponds to $\alpha = 0.64$. This somewhat large value of α may correlate with the observation that the $\pi^- p \rightarrow \pi\pi N$ reaction is dominated near threshold by formation of the Roper resonance. [In general, $\Gamma(W)$ is expected to increase relatively fast in the vicinity of an important inelastic threshold.] Recently, Morsch *et al.* studied the Roper resonance by α -proton scattering [15]. Their production experiment led to a slightly low mass of about 1400 MeV and a rather narrow width of about 160–170 MeV. The apparently contradictory results between formation and production experiments may be understood if the parameters from the α -proton experiment can be identified with the pole pa-

rameters M_0 and Γ_0 . As discussed below, estimates of M_0 and Γ_0 from the πN experiments yield the mean values, 1372 ± 11 MeV and 213 ± 48 MeV, in reasonable agreement with the results of Morsch *et al.*

From Table II, it is also clear that three analyses of resonance formation in πN scattering experiments indicate that the $N(1535)\frac{1}{2}^-$ is narrow, in disagreement with the CMB80 result. The mass and width of this resonance are nontrivial to determine because it has a large branching fraction (about 50%) to ηN , and the threshold for η production is at 1488 MeV. Recently, Clajus and Nefkens determined the mass and width of this resonance by using a Breit-Wigner term to fit available total cross-section data for the $\pi^- p \rightarrow \eta n$ reaction near threshold [16]. The mass and width obtained from their fit were $M_0 = 1483 \pm 16$ MeV and $\Gamma_0 = 204 \pm 21$ MeV, respectively. As noted by Clajus and Nefkens, this mass is well below the (Breit-Wigner) value recommended for the $N(1535)\frac{1}{2}^-$. This seemingly contradictory result is easily explained by noting that Clajus and Nefkens used a *constant* width in their Breit-Wigner fit; hence, their values *must* be interpreted as the physical mass M_0 and width Γ_0 associated with the pole position. Their values then agree with the mean pole values of $M_0 = 1502 \pm 12$ MeV and $\Gamma_0 = 169 \pm 80$ MeV (see below).

TABLE II. Conventional Breit-Wigner parameters for $I = \frac{1}{2}$ resonances as determined from four analyses: KSU92 [6]; VPI91 [10]; CMB80 [9]; and KH80 [12]. The first entry lists M and the second lists Γ .

Resonance	KSU92	VPI91 ^a	CMB80	KH80	Mean
$N(1440)\frac{1}{2}^+$	1462 391	1451 383	1440 340	1410 135	1441 ± 22 312 ± 120
$N(1520)\frac{3}{2}^-$	1524 124	1525 115	1525 120	1519 114	1523 ± 3 118 ± 5
$N(1535)\frac{1}{2}^-$	1534 151	1516 120	1550 240	1526 120	1532 ± 14 158 ± 57
$N(1650)\frac{1}{2}^-$	1659 173	1668 163	1650 150	1670 180	1662 ± 9 167 ± 13
$N(1675)\frac{5}{2}^-$	1676 159	1668 130	1675 160	1679 120	1675 ± 5 142 ± 20
$N(1680)\frac{5}{2}^+$	1684 139	1684 122	1680 120	1684 128	1683 ± 2 127 ± 9
$N(1710)\frac{1}{2}^+$	1717 480	1704 578	1700 90	1723 120	1711 ± 11 317 ± 248
$N(1720)\frac{3}{2}^+$	1717 380	1694 127	1700 125	1710 190	1705 ± 10 206 ± 120
$N(2190)\frac{7}{2}^-$	2127 550	2176 580	2200 500	2140 390	2161 ± 33 505 ± 83

^aCalculated from the pole positions (solution SM90) in Ref. [10]. Parameters in the table for $N(1440)\frac{1}{2}^+$ were determined from the pole of most physical significance at $(1360 - i 126)$ MeV. An auxiliary pole was found at $(1413 - i 128)$ MeV (see text).

The Breit-Wigner width parameters for the $N(1710)\frac{1}{2}^+$ and the $N(1720)\frac{3}{2}^+$ are not determined consistently among the four analyses; this lack of agreement is understandable given that these two resonances are very inelastic.

From Table III, it can be noted that the calculated mass parameter of 1670 MeV for the $\Delta(1600)\frac{3}{2}^+$ (determined from the VPI91 pole position) agrees better with the KSU92 value of 1706 MeV than with the CMB80 and KH80 values of 1600 and 1522 MeV, respectively. A higher mass near 1700 MeV is in better agreement with quark-model predictions [17,18].

Finally, Table III shows that the calculated mass parameter of 2095 MeV for the $\Delta(1930)\frac{5}{2}^-$ (determined from the VPI91 pole position) is about 150 MeV higher than values of the three other analyses. The lower values have presented long-standing problems for quark-model calculations [18].

III. SUMMARY AND CONCLUSIONS

A simple approximation was derived to relate the pole position in a resonant scattering amplitude to the conventional Breit-Wigner parameters, M and Γ . The physical mass M_0 is defined as the real part of the pole position and the physical width Γ_0 is -2 times the imaginary part of the pole position [1-4]. On general grounds, it is expected that $M > M_0$ and $\Gamma > \Gamma_0$.

Tables IV and V compare the physical mass M_0 and width Γ_0 determined from the real and imaginary parts of

the pole positions from four analyses, including the 1992 Kent State University analysis (KSU92) [6], the 1991 Virginia Polytechnic Institute and State University analysis (VPI91) [10], the 1980 Carnegie-Mellon-Berkeley analysis (CMB80) [9], and the 1980 Karlsruhe-Helsinki analysis (KH80) [12]. The tables also give unweighted means with sample standard deviations. The pole parameters listed under the heading KSU92 were estimated from the conventional Breit-Wigner parameters in Ref. [6] using values of α from Table I, and the pole parameters listed under the heading KH80 are from the recent speed-plot fits by Höhler [14]. Contrary to expectations, Höhler's recent determinations of pole parameters for several resonances do *not* satisfy the condition $\Gamma > \Gamma_0$ when compared with the early KH80 width parameters; consequently, the early KH80 width parameters may be underestimated in some cases (the Roper resonance, for example). This is an important point because results of that analysis have historically provided a main contribution for the estimation of standard resonance parameters, as quoted, for example, by the Particle Data Group [19].

My relationship between the pole position and Breit-Wigner parameters was tested by using results from the Carnegie-Mellon-Berkeley (CMB) partial-wave analysis of πN scattering [9]. The relationship was then used to calculate conventional Breit-Wigner parameters from the pole positions determined in a recent πN partial-wave analysis by the Virginia Tech (VPI) group [10]. For most cases, the parameters derived from the VPI pole positions agree very well with the results of a recent analysis performed at Kent State University (KSU) [6]. There

TABLE III. Conventional Breit-Wigner parameters as in Table II but for $I = \frac{3}{2}$ resonances.

Resonance	KSU92	VPI91 ^a	CMB80	KH80	Mean
$\Delta(1232)\frac{3}{2}^+$	1231	1232	1232	1233	1232 ± 1
	118	119	120	116	118 ± 2
$\Delta(1600)\frac{3}{2}^+$	1706	1670	1600	1522	1625 ± 81
	430	288	300	220	310 ± 88
$\Delta(1620)\frac{1}{2}^-$	1672	1607	1620	1610	1627 ± 30
	154	133	140	139	142 ± 9
$\Delta(1700)\frac{3}{2}^-$	1762	1679	1710	1680	1690 ± 18^b
	600	229	280	230	246 ± 29^b
$\Delta(1905)\frac{5}{2}^+$	1881	1860	1910	1905	1889 ± 23
	327	305	400	260	323 ± 58
$\Delta(1910)\frac{1}{2}^+$	1882	2010	1910	1888	1923 ± 60
	239	434	225	280	295 ± 96
$\Delta(1930)\frac{5}{2}^-$	1956	2095	1940	1901	1973 ± 85
	530	457	320	195	376 ± 149
$\Delta(1950)\frac{7}{2}^+$	1945	1939	1950	1913	1937 ± 16
	300	289	340	224	288 ± 48

^aCalculated from the pole positions (solution SM90) in Ref. [10].

^bWithout KSU92.

TABLE IV. Pole parameters for $I = \frac{1}{2}$ resonances as determined from four analyses: KSU92 [6]; VPI91 [10]; CMB80 [9]; and KH80 [12,14]. The first entry lists the physical mass M_0 and the second lists the physical width Γ_0 .

Resonance	KSU92 ^a	VPI91	CMB80	KH80	Mean
$N(1440)\frac{1}{2}^+$	1369 257	1360 252	1375 180	1385 164	1372 ± 11 213 ± 48
$N(1520)\frac{3}{2}^-$	1509 116	1511 108	1510 114	1510 120	1510 ± 1 115 ± 5
$N(1535)\frac{1}{2}^-$	1513 138	1499 110	1510 260	1487	1502 ± 12 169 ± 80
$N(1650)\frac{1}{2}^-$	1648 170	1657 160	1640 150	1670 163	1654 ± 13 161 ± 8
$N(1675)\frac{5}{2}^-$	1660 152	1655 124	1660 140	1656 126	1658 ± 3 136 ± 13
$N(1680)\frac{5}{2}^+$	1668 132	1670 116	1667 110	1673 135	1670 ± 3 123 ± 12
$N(1710)\frac{1}{2}^+$	1661 452	1636 544	1690 80	1690 200	1669 ± 26 319 ± 216
$N(1720)\frac{3}{2}^+$	1660 342	1675 114	1680 120	1686 187	1675 ± 11 191 ± 106
$N(2190)\frac{7}{2}^-$	2017 440	2060 464	2100 400	2042 482	2055 ± 35 447 ± 35

^aCalculated from the Breit-Wigner parameters in Ref. [6].

TABLE V. Pole parameters as in Table IV but for $I = \frac{3}{2}$ resonances.

Resonance	KSU92 ^a	VPI91	CMB80	KH80	Mean
$\Delta(1232)\frac{3}{2}^+$	1211 ^b 102 ^b	1210 100	1210 100	1209 100	1210 ± 1 100 ± 1
$\Delta(1600)\frac{3}{2}^+$	1620 344	1612 230	1550 200	1550	1583 ± 38 258 ± 76
$\Delta(1620)\frac{1}{2}^-$	1649 139	1587 120	1600 120	1608 116	1611 ± 27 124 ± 10
$\Delta(1700)\frac{3}{2}^-$	1675 545	1646 208	1675 220	1651 159	1657 ± 16^c 196 ± 32^c
$\Delta(1905)\frac{5}{2}^+$	1811 247	1794 230	1890 260	1829 303	1831 ± 42 260 ± 31
$\Delta(1910)\frac{1}{2}^+$	1849 219	1950 398	1880 200	1874 283	1888 ± 43 275 ± 89
$\Delta(1930)\frac{5}{2}^-$	1867 462	2018 398	1890 260	1850 180	1906 ± 76 325 ± 128
$\Delta(1950)\frac{7}{2}^+$	1888 247	1884 238	1890 260	1878 230	1885 ± 5 244 ± 13

^aCalculated from the Breit-Wigner parameters in Ref. [6].

^bCalculated with $\alpha = 0.40$, as discussed in the text.

^cWithout KSU92.

were, however, three notable exceptions: the $N(1720)\frac{3}{2}^+$ was found to have a significantly narrower width than determined in the KSU analysis, and the $\Delta(1910)\frac{1}{2}^+$ and $\Delta(1930)\frac{5}{2}^-$ were found to have masses over 100 MeV higher than in the KSU analysis. These differences possibly arise because these resonances couple weakly to the πN channel; therefore, they are not excited strongly in elastic πN scattering.

My relationship was also used to estimate pole parameters from the KSU Breit-Wigner parameters [6]. These were compared with pole parameters from the CMB analysis [9], the VPI analysis [10], and from the recent work by Höhler [14], who fitted speed plots of amplitudes from the KH80 analysis [12]. Mean values for M_0 and Γ_0 were determined from these four sets of pole parameters. The standard deviations among the pole values for a given resonance are generally smaller than the corresponding standard deviations among the Breit-Wigner parameters. The four sets agree particularly well for the pole mass and width of the $\Delta(1232)\frac{3}{2}^+$, $N(1520)\frac{3}{2}^-$, $N(1650)\frac{1}{2}^-$, $N(1675)\frac{5}{2}^-$, $N(1680)\frac{5}{2}^+$, and $\Delta(1950)\frac{7}{2}^+$. This stability in the determination of the pole parameters for resonances with large πN couplings provides further evidence that these parameters are less model dependent than the corresponding Breit-Wigner parameters, as noted in the Introduction.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation under Grant No. PHY-9106512.

APPENDIX: MODEL DEPENDENCY OF BREIT-WIGNER PARAMETERS

If one assumes a “relativistic” Breit-Wigner form, rather than Eq. (1), the partial-wave T -matrix amplitude in the vicinity of a pole becomes

$$T \approx \frac{g(W)}{M^2 - W^2 - iM\Gamma(W)}. \quad (\text{A1})$$

For $M \gg \Gamma/2$, a similar derivation to that presented in Sec. I yields the relationship

$$M_0 \approx M - \frac{\Gamma}{2} \left(\frac{\alpha}{1 + \alpha^2} \right) - \frac{\Gamma^2/(8M)}{1 + \alpha^2}, \quad (\text{A2a})$$

$$\Gamma_0 \approx \frac{\Gamma}{1 + \alpha^2} - \frac{\Gamma^2}{4M} \left(\frac{\alpha}{1 + \alpha^2} \right), \quad (\text{A2b})$$

where here $\alpha = \Gamma'/2 - \Gamma/(2M)$. The last terms on the right-hand side (RHS) of the expressions in Eq. (A2) are generally very small. Clearly these expressions for M_0 and Γ_0 reduce to the results in Sec. I when $\Gamma/M \ll \Gamma'$. In that situation, the differences in fitted Breit-Wigner parameters arising from the use of either relativistic or nonrelativistic models are expected to be small.

Finally, it is noteworthy that other versions of the Breit-Wigner formula are in common use. For example, in fitting the e^+e^- cross section near the Z -boson resonance, it is common to represent the Z -boson propagator by a term proportional to [3]

$$T \approx \frac{g(W)}{M^2 - W^2 - iW^2\Gamma/M}, \quad (\text{A3})$$

where the conventional Breit-Wigner parameters M and Γ are constants. This equation may be related to Eq. (A1) by making the identification, $\Gamma(W) = (W/M)^2\Gamma$, which implies that in Eq. (A2), $\alpha = \Gamma/(2M)$.

-
- [1] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992), p. VIII.10.
- [2] G. Dillon, Europhys. Lett. **20**, 389 (1992).
- [3] S. Willenbrock and G. Valencia, Phys. Lett. B **259**, 373 (1991); R. Stuart, *ibid.* **262**, 113 (1991); A. Sirlin, Phys. Rev. Lett. **67**, 2127 (1991).
- [4] D. B. Lichtenberg, Phys. Rev. D **10**, 3865 (1974).
- [5] The function $D(W)$ is assumed to be analytic at $W = W_p$, which is a zero of the function. Since the zeros of an analytic function are isolated, there is a neighborhood about $W = W_p$ throughout which $D(W)$ has no other zeros; in this region, $D(W)$ is continuous and its derivative exists. Thus, the method used to estimate W_p is expected to be reliable, even if there exists a nearby branch cut in the T -matrix amplitude.
- [6] D. M. Manley and E. M. Saleski, Phys. Rev. D **45**, 4002 (1992).
- [7] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Springer-Verlag, New York, 1979).
- [8] D. M. Manley, R. A. Arndt, Y. Goradia, and V. L. Teplitz, Phys. Rev. D **30**, 904 (1984).
- [9] R. E. Cutkosky, C. P. Forsyth, R. E. Hendrick, and R. L. Kelly, Phys. Rev. D **20**, 2839 (1979); R. E. Cutkosky, R. E. Hendrick, J. W. Alcock, Y. A. Chao, R. G. Lipes, J. C. Sandusky, and R. L. Kelly, *ibid.* **20**, 2804 (1979); R. E. Cutkosky, in *Baryon 1980*, Proceedings of the IVth International Conference on Baryon Resonances, Toronto, 1980, edited by N. Isgur (University of Toronto, Toronto, 1980), p. 19.
- [10] R. A. Arndt, Z. Li, L. D. Roper, R. L. Workman, and J. M. Ford, Phys. Rev. D **43**, 2131 (1991).
- [11] R. A. Arndt, R. L. Workman, and M. M. Pavan, Phys. Rev. C **49**, 2729 (1994).
- [12] G. Höhler, F. Kaiser, R. Koch, and E. Pietarinen, *Handbook of Pion-Nucleon Scattering* [Physics Data No. 12-1 (1979)]; G. Höhler, in *Pion-Nucleon Scattering*, edited by H. Schopper, Landolt-Börnstein, New Series, Group 1, Vol. 9, Pt. b (Springer, New York, 1983); R. Koch, in *Baryon 1980* [9], p. 3; R. Koch and E. Pietarinen, Nucl. Phys. **A336**, 331 (1980).

- [13] R. E. Cutkosky and S. Wang, *Phys. Rev. D* **42**, 235 (1990).
- [14] G. Höhler, in πN Newsletter No. 9 (Proceedings of the 5th International Symposium on Meson-Nucleon Physics and the Structure of the Nucleon, Volume II), p. 1.
- [15] H. P. Morsch, M. Boivin, W. Jacobs, F. Plouin, J. Yonnet, P. Zupranski, R. Frascaria, R. Siebert, E. Warde, J. P. Didelez, B. Saghai, and P. E. Tégner, *Phys. Rev. Lett.* **69**, 1336 (1992).
- [16] M. Clajus and B. M. K. Nefkens, in πN Newsletter No. 7 (1992), p. 76.
- [17] N. Isgur and G. Karl, *Phys. Rev. D* **19**, 2653 (1979); S. Capstick and N. Isgur, *ibid.* **34**, 2809 (1986).
- [18] C. P. Forsyth and R. E. Cutkosky, *Z. Phys. C* **18**, 219 (1983).
- [19] Particle Data Group, L. Montanet *et al.*, *Phys. Rev. D* **50**, 1173 (1994).