Model Dependence of Nucleon Resonance Parameters for $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$

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with participation of:

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- Introduction:

  Result from BRAG 2001 on $P_{33}(1232)$

- Definition of Photon Couplings or Helicity Amplitudes:

  $A_{1/2}$, $A_{3/2}$, $S_{1/2}$

- Comparison with Partial Wave Analyses from:

  GWU/SAID, MAID,

  CC/Bennhold and UIM,DR/Aznauryan

- Preliminary Conclusions

Result of BRAG 2001
on Multipole Analysis in the Delta Region
with Benchmark Dataset

summarized by R. Davidson, Proc. NSTAR2001 in Mainz, p. 204

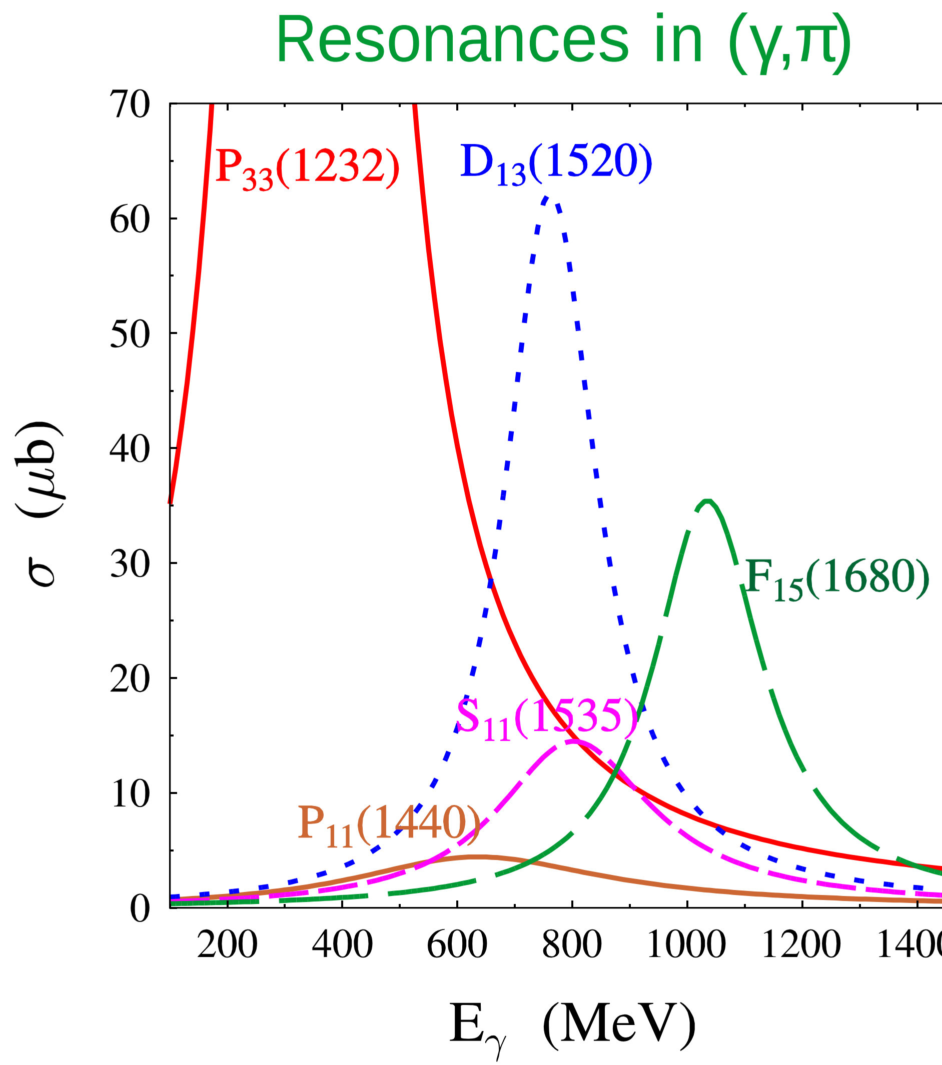
	M1	E2	E2/M1 (%)
RPI	286	-7.2	-2.55
GWU	281	-7.2	-2.57
HA	281	-6.6	-2.35
MAID	275	-5.3	-1.93
KY	280	-6.2	-2.24
AZ	278	-6.3	-2.28
OM	288	-7.8	-2.77
AVG	281.3 ± 4.5	-6.6 ± 0.8	-2.38 ± 0.27

main conclusion:

model error in M1 coupling: ~ 2%

model error in E2 and E/M ~ 12%

Prominent Resonances in Pion Photoproduction



problems to face:

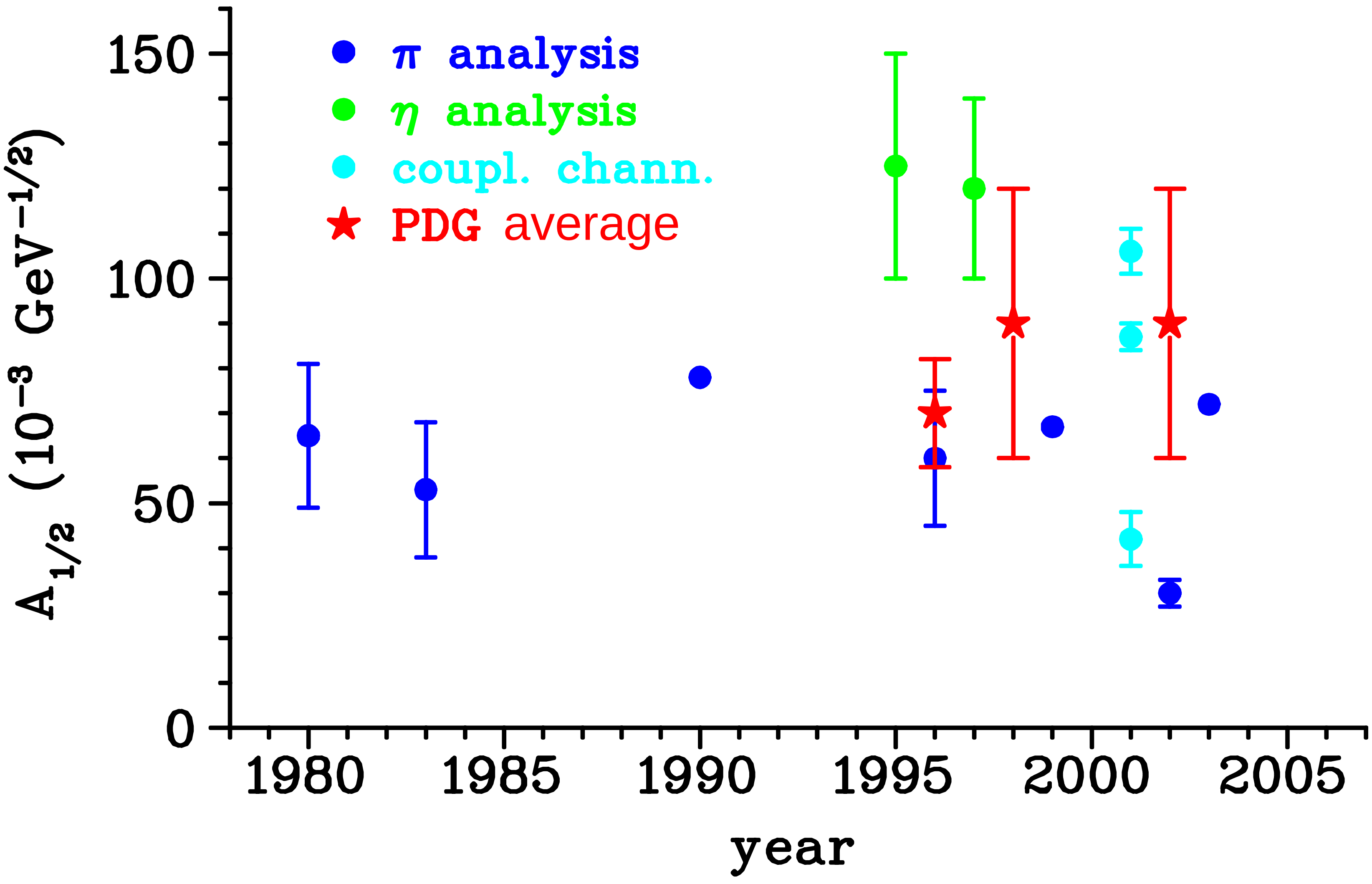
- we need very precise partial wave amplitudes
  this is in principle possible, we are on the way
- we need to separate resonance and background
  this is in principle impossible, but it could work approximately
- we need to know precisely the mass, width and single-pion branching ratio
  this could be improved (in principle), very important for the width

Particle Data Group 2002

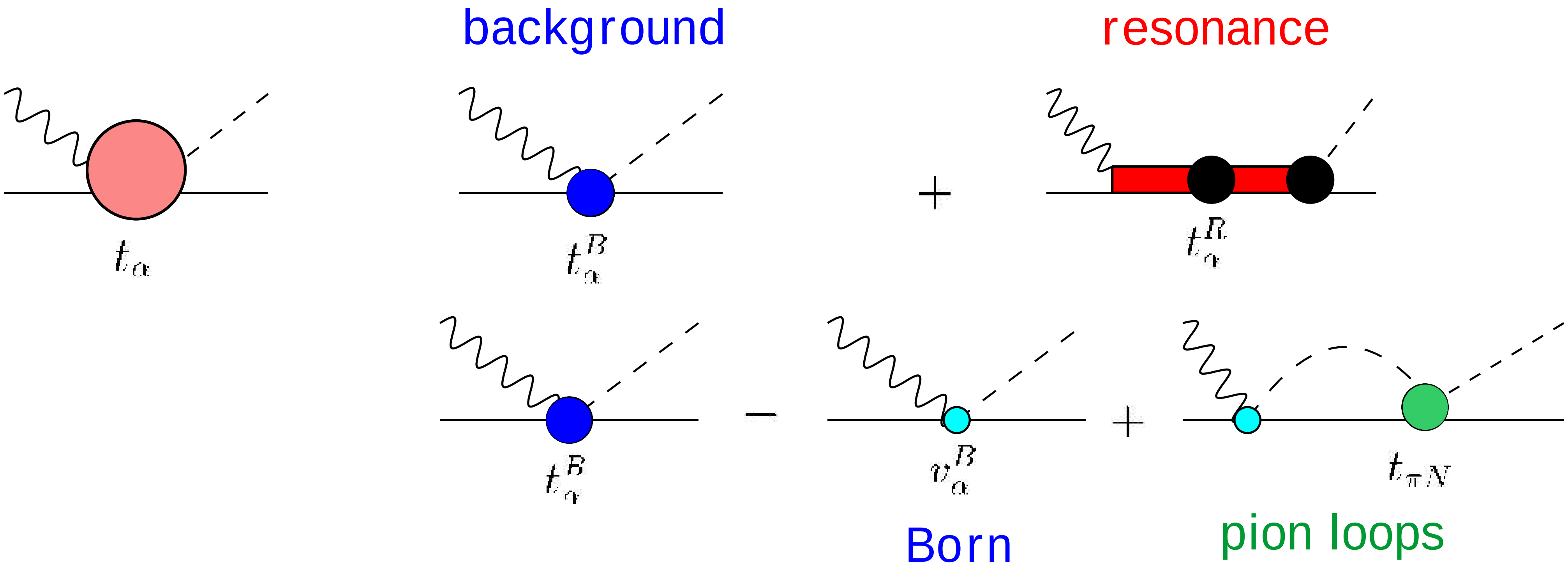
	M_R	Γ_R	$\beta_\pi = \frac{\Gamma_\pi}{\Gamma_R}$	${}_pA_{1/2}$	${}_pA_{3/2}$
$P_{11}(1440)$	1440^{+30}_{-10}	350 ± 100	$.65 \pm .05$	-65 ± 4	–
$D_{13}(1520)$	1520^{+10}_{-5}	120^{+15}_{-10}	$.55 \pm .05$	-24 ± 9	166 ± 5
$S_{11}(1535)$	1535^{+20}_{-15}	150^{+100}_{-50}	$.45 \pm .10$	90 ± 30	–

Photon Decay Amplitudes

$S_{11}(1535) \longrightarrow p\,\gamma$



Dynamical Model Picture



the background is not only Born

the resonances are dressed

comparison with 3 methods for resonance - background separation

method a) used in SM02 by GWU/SAID group
b) used in previous SAID analyses
c) our (MAID) analysis

the partial wave amplitudes are fitted with a relatively simple form (~5-7 par.)
of smoth background plus Breit-Wigner resonance shape
in an energy region around the resonance position

$$a) \quad T_a = (1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N} + (\mathrm{C} + i\,\mathrm{D})(\mathrm{Im}\, t_{\pi N} - |t_{\pi N}|^2)$$

$$b) \quad T_b = [(1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N}]\, e^{i\phi}$$

$$c) \quad T_c = (1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N}\, e^{i\phi}$$

in details:

$$a)\quad T_a = (1+\mathrm{i}\,t_{\pi N})(Born+\mathbf{A})+\mathbf{R}\,t_{\pi N}+(\mathbf{C}+i\,\mathbf{D})(\mathrm{Im}\,t_{\pi N}-|t_{\pi N}|^2)$$

$$b)\quad T_b = [(1+\mathrm{i}\,t_{\pi N})(Born+\mathbf{A})+\mathbf{R}\,t_{\pi N}]\,e^{i\phi}$$

$$c)\quad T_c = (1+\mathrm{i}\,t_{\pi N})(Born+\mathbf{A})+\mathbf{R}\,t_{\pi N}\,e^{i\phi}$$

$$\mathrm{Im}\,t_{\pi N}-|t_{\pi N}|^2 = (1-\eta^2)/4 \quad (=0,\ \text{below } 2\pi \text{ threshold})$$

$$t_{\pi N} = \frac{M_R\Gamma_R}{M_R^2-W^2-i\,M_R\Gamma_{tot}(W)}\sqrt{\frac{\Gamma_\pi(W)}{\Gamma_\pi(M_R)}} \stackrel{W=M_R}{=} i$$

energy dependent widths (including cusp effects)

$$\Gamma_{tot}(W) = \Gamma_\pi(W)+\Gamma_{2\pi}(W)+\Gamma_\eta(W)$$

$$\Gamma_{tot}(M_R) = \Gamma_R$$

up to 7 fit parameters:

$$\begin{aligned} \mathrm{A} &= (a_1+a_2E \\ \phi &= (b_1+b_2E)(\mathrm{Im}\,t_{\pi N}-|t_{\pi N}|^2) \\ R &= r_1 \\ C &= c_1 \\ D &= d_1 \end{aligned}$$

for P_{11} and S_{11} also mass and width:

$M_R\,,\ \Gamma_R$

reduced resonance multipoles:

$$\bar{\mathcal{A}}_\alpha(Q^2) = \frac{1}{c_{\pi N} f_{\pi R}(M_R)} \mathrm{Im}\mathcal{A}_\alpha^{res}(M_R, Q^2)$$

kinematical factor

$$f_{\pi R}(M_R) = \left[\frac{1}{(2j+1)\pi} \frac{k_W}{|q|} \frac{m_N}{M_R} \frac{\Gamma_\pi}{\Gamma_R^2}\right]^{1/2}$$

isospin factor

$$c_{\pi N} = \begin{cases} -1/\sqrt{3} & : \quad I = 1/2 \\ \sqrt{3/2} & : \quad I = 3/2 \end{cases}$$

photon couplings $(Q^2 = 0)$:

$$\begin{aligned}
S_{11} &: \quad A_{1/2} = -\bar{E}_{0+} \\
P_{11} &: \quad A_{1/2} = \bar{M}_{1-} \\
D_{13} &: \quad A_{1/2} = -\frac{1}{2}(\bar{E}_{2-} - 3\bar{M}_{2-}) \\
&\quad\;\; A_{3/2} = -\frac{\sqrt{3}}{2}(\bar{E}_{2-} + \bar{M}_{2-})
\end{aligned}$$

e.g. for Roper resonance $P_{11}(1440)$ we get:

$$A_{1/2} = -\sqrt{\frac{6\pi q_\pi^R \, M_R \, \Gamma_R}{k_\gamma^R \, M_N \, \Gamma_\pi / \Gamma_R}} \,\mathrm{Im}\, M_{1-}^{res}(W = M_R)$$

Definition of Helicity Amplitudes or Photon Couplings

for resonance excitation: γ + N -> R

$$A_{1/2} = -\sqrt{\frac{2\pi\alpha_{f.s.}}{k_w}}\left\langle R, \tfrac{1}{2}\middle| J_+ \middle| N, -\tfrac{1}{2}\right\rangle \xi \qquad A_{3/2} = -\sqrt{\frac{2\pi\alpha_{f.s.}}{k_w}}\left\langle R, \tfrac{3}{2}\middle| J_+ \middle| N, \tfrac{1}{2}\right\rangle \xi$$

ξ is a phase, which depends on the pion-decay matrix element
the photon couplings are real numbers

for $P_{11}(1440)$ in the simple harmonic oscillator CQM:

$$A_{1/2} = -\sqrt{\frac{\pi\alpha_{f.s.}}{3k_w}}\frac{\mu_p}{6M_N}\alpha_0\left(\frac{q^2}{\alpha_0^2}\right)^3 e^{-q^2/6\alpha_0^2} \quad \alpha_0 = 0.41 GeV \text{ and } Q^2 = 0 \longrightarrow -24 \; 10^{-3} GeV^{-1/2}$$

S_{11} : $A_{1/2} = 168$

D_{13} : $A_{1/2} = -23$

$A_{3/2} = 125$

Results of our Analysis

$$a) \quad T_a = (1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N} + (\mathrm{C} + i\,\mathrm{D})(\mathrm{Im}\, t_{\pi N} - |t_{\pi N}|^2)$$

$$b) \quad T_b = [(1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N}]\, e^{i\phi}$$

$$c) \quad T_c = (1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N}\, e^{i\phi}$$

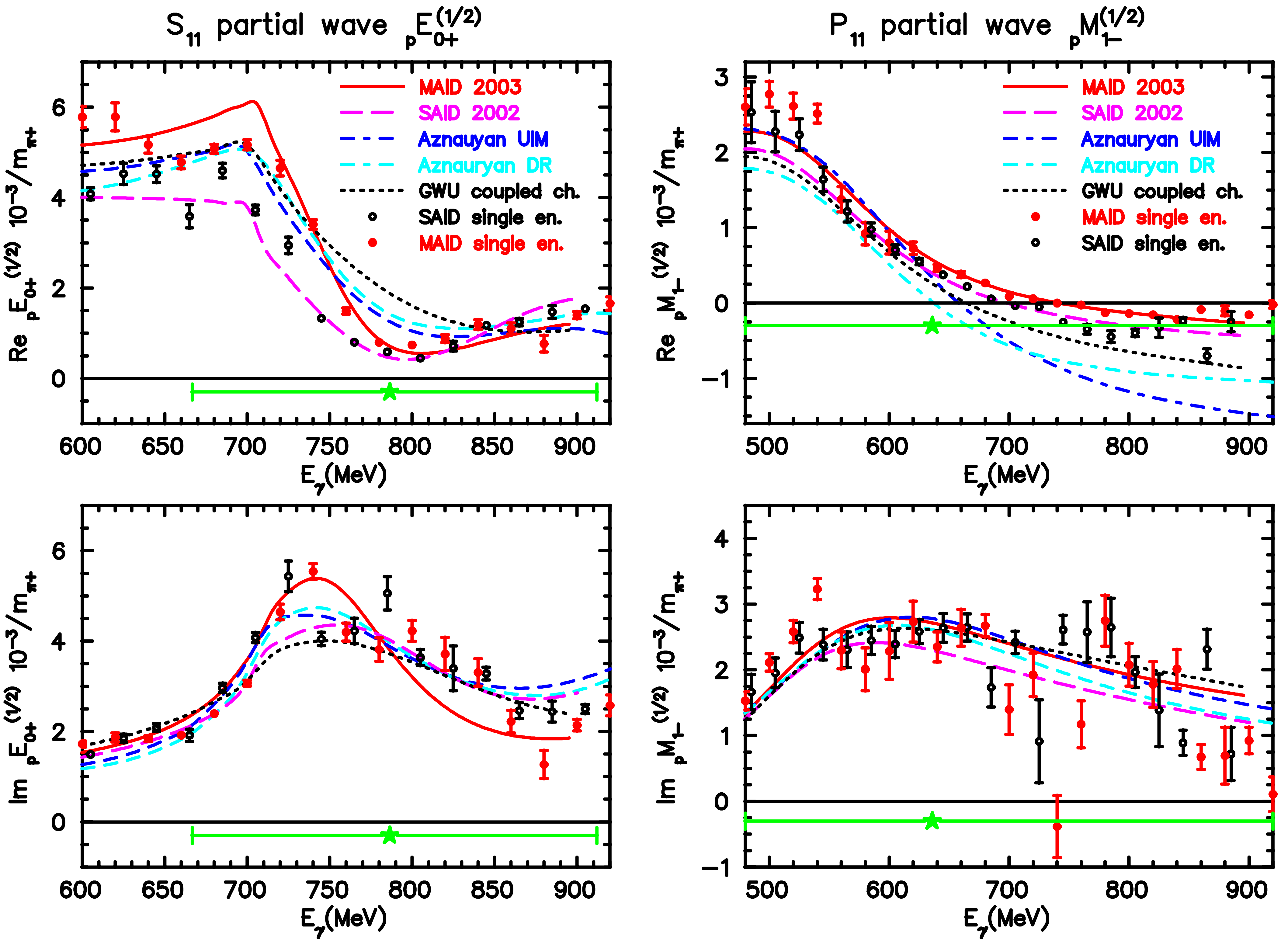
- most solutions can be fitted well with all 3 methods, only SAID solutions for S11 is better fitted by method a)
- method b) and c) are very similar
  we will show only results of method c) and compare it with a)
- most problematic case is the S11 partial wave
- less problematic, but with large experimental uncertainty is P11 pw
- no problems with the strong D13 pw

we have applied our 3 methods
to the partial wave analyses from different groups:

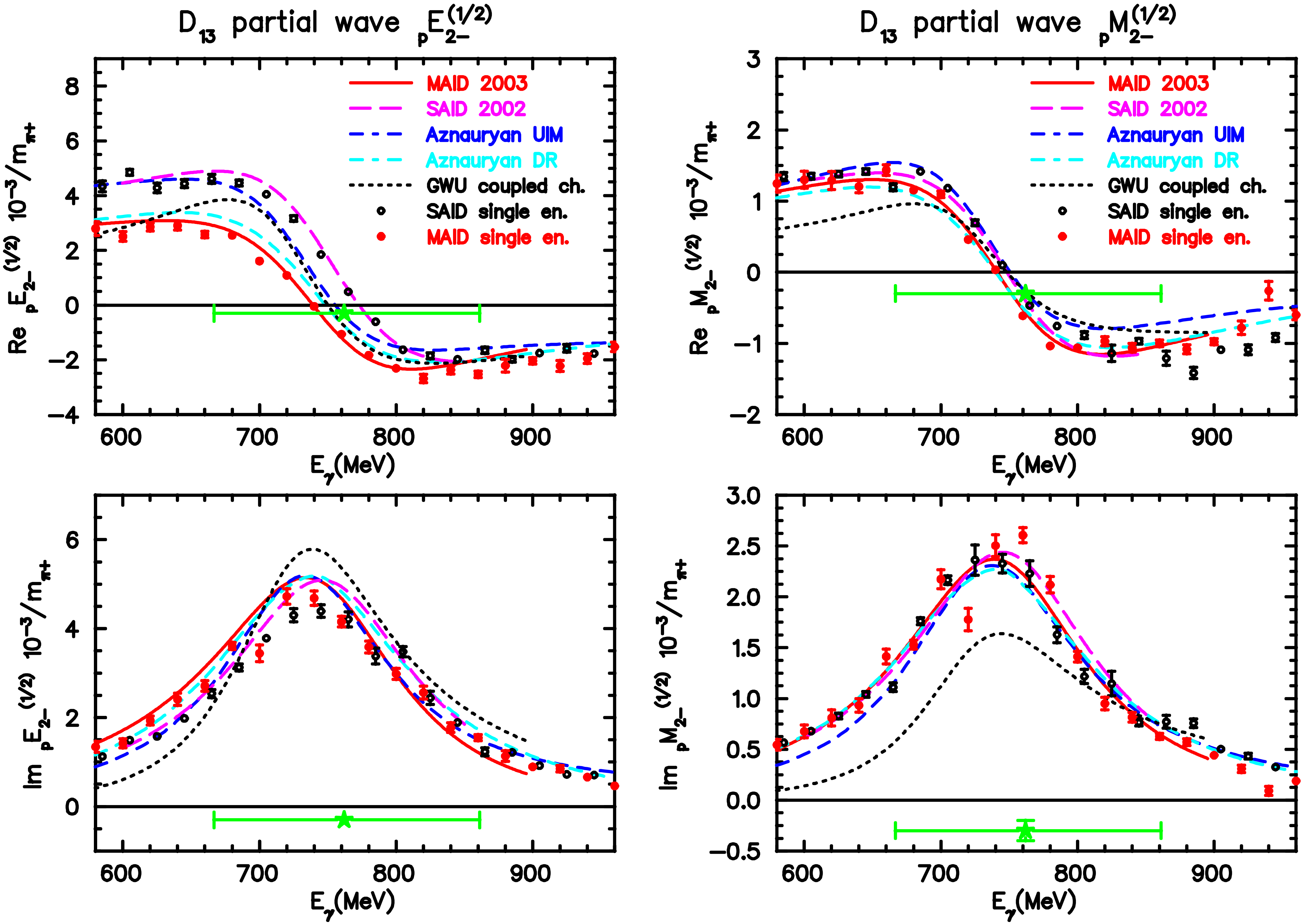
- MAID global solution MAID2003
- MAID single-energy solution MAIDse04
- SAID/GWU global solution SM02
- SAID/GWU single-energy solution SE02
- Bennhold/GWU coupled channels GWU/CC
- Aznauryan/JLab reggeized unitary isobar model AZ-UIM
- Aznauryan isobar model with dispersion relations AZ-DR

and we also compare to the following published results:

- PDG values
- GWU/SAID analysis SM02, PR C 66,055213 (2002)
- γ,η analysis for S_{11} by Krusche, Schadmand, review 2003



S_{11} partial wave $_pE_{0+}^{(1/2)}$
P_{11} partial wave $_pM_{1-}^{(1/2)}$
MAID 2003
SAID 2002
Aznauyan UIM
Aznauryan DR
GWU coupled ch.
SAID single en.
MAID single en.
Re $_pE_{0+}^{(1/2)}$ $10^{-3}/m_{\pi+}$
Re $_pM_{1-}^{(1/2)}$ $10^{-3}/m_{\pi+}$
Im $_pE_{0+}^{(1/2)}$ $10^{-3}/m_{\pi+}$
Im $_pM_{1-}^{(1/2)}$ $10^{-3}/m_{\pi+}$
E_γ(MeV)

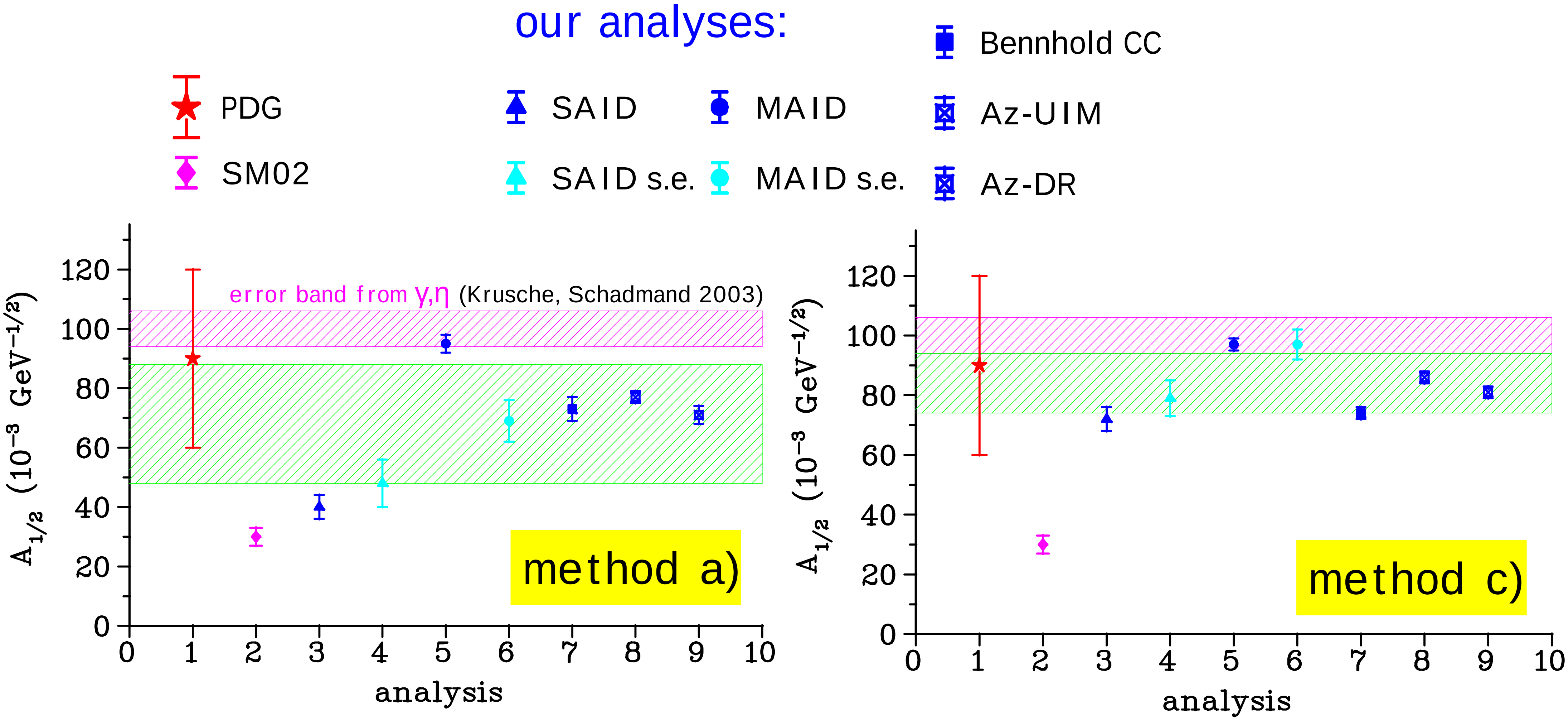


D13 partial wave pE2−(1/2)
D13 partial wave pM2−(1/2)
MAID 2003
SAID 2002
Aznauryan UIM
Aznauryan DR
GWU coupled ch.
SAID single en.
MAID single en.
Re pE2−(1/2) 10−3/mπ+
Re pM2−(1/2) 10−3/mπ+
Im pE2−(1/2) 10−3/mπ+
Im pM2−(1/2) 10−3/mπ+
Eγ(MeV)

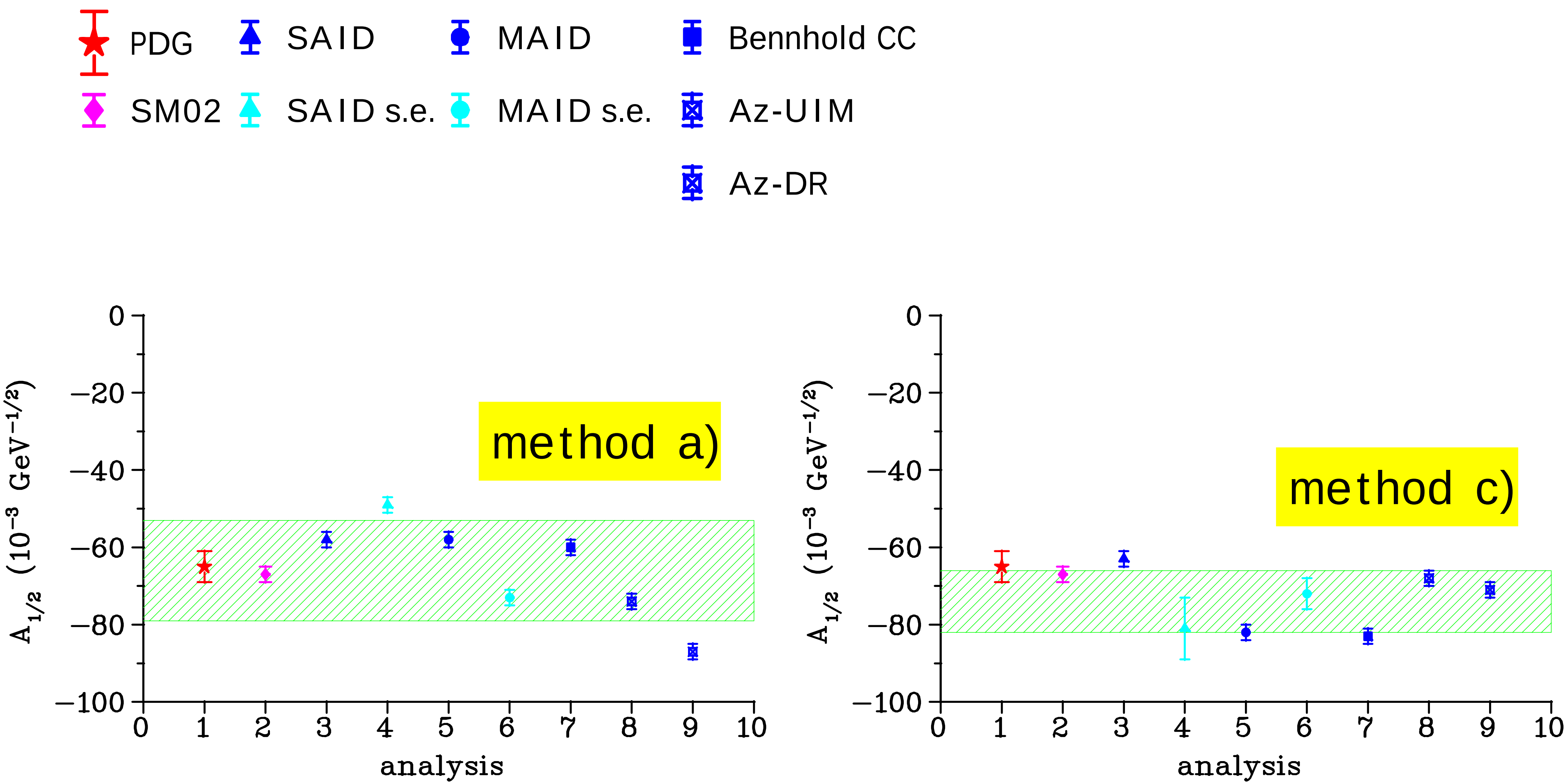
$S_{11}(1535)$

method $a)$ $\quad T_a = (1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N} + (\mathrm{C} + i\,\mathrm{D})(\mathrm{Im}\, t_{\pi N} - |t_{\pi N}|^2)$

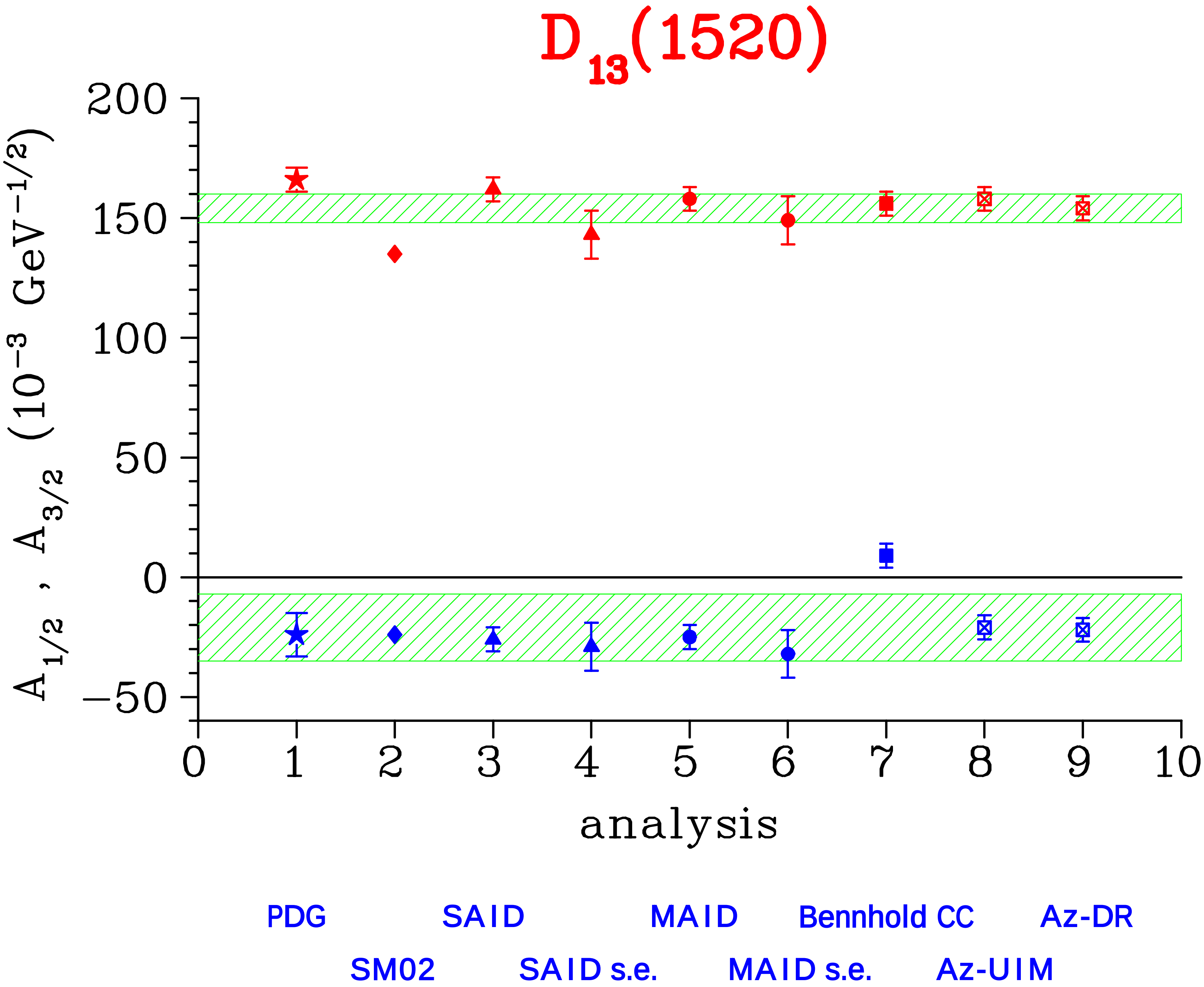
method $c)$ $\quad T_c = (1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N}\, e^{i\phi}$



$P_{11}(1440)$



PDG
SAID
MAID
Bennhold CC
SM02
SAID s.e.
MAID s.e.
Az-UIM
Az-DR
method a)
method c)
A1/2 (10−3 GeV−1/2)
analysis
0
−20
−40
−60
−80
−100
0 1 2 3 4 5 6 7 8 9 10



D13(1520)
200
150
100
50
0
−50
A1/2 , A3/2 (10−3 GeV−1/2)
0
1
2
3
4
5
6
7
8
9
10
analysis
PDG
SAID
MAID
Bennhold CC
Az-DR
SM02
SAID s.e.
MAID s.e.
Az-UIM

(Preliminary) Conclusions

- method a) is problematic, even if it fits some SAID multipoles better,
  it can mix background and resonance
  method b) is similar to c)
  we generally propose method c)

$$T_c \; = \; (1 + \mathrm{i}\, t_{\pi N})(Born + \mathrm{A}) + \mathrm{R}\, t_{\pi N}\, e^{i\phi}$$

- it is a good idea to use normalized photon couplings with average values for $M_R\,, \Gamma_R\,, \beta_\pi$ from PDG
- remaining uncertainties are:
  a) from experimental data
  b) from pw analysis
  c) from energy range used in the BW fits